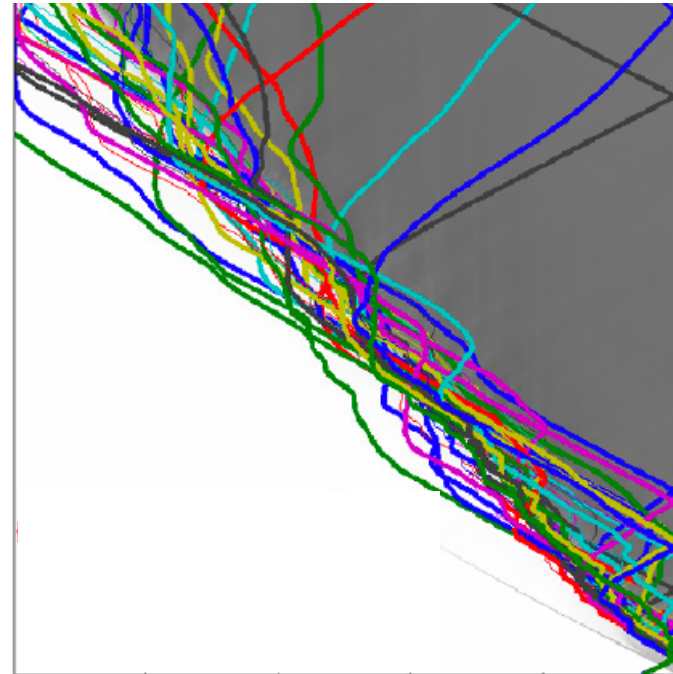
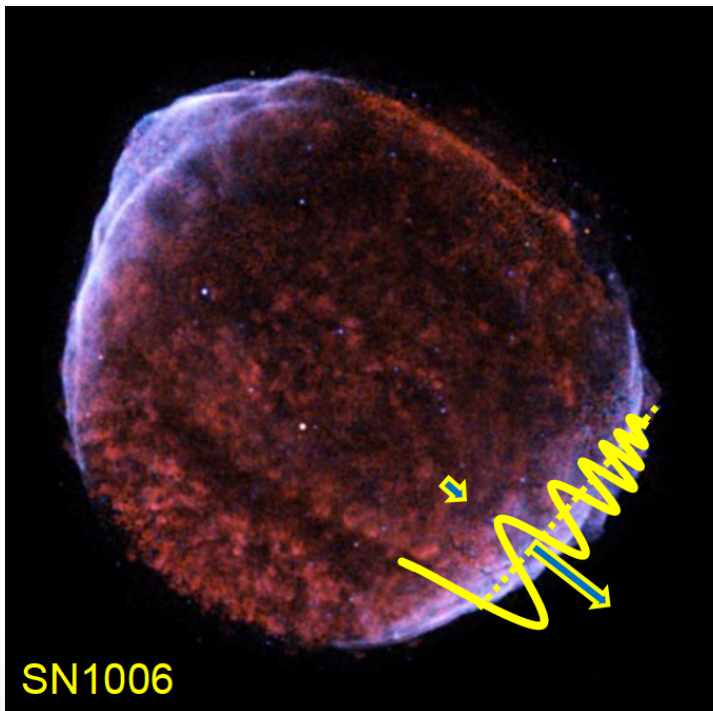

Particle acceleration in astrophysics: Fermi acceleration

Illya Plotnikov
Ateliers Astroplasma IRAP, 20 Jan 2021

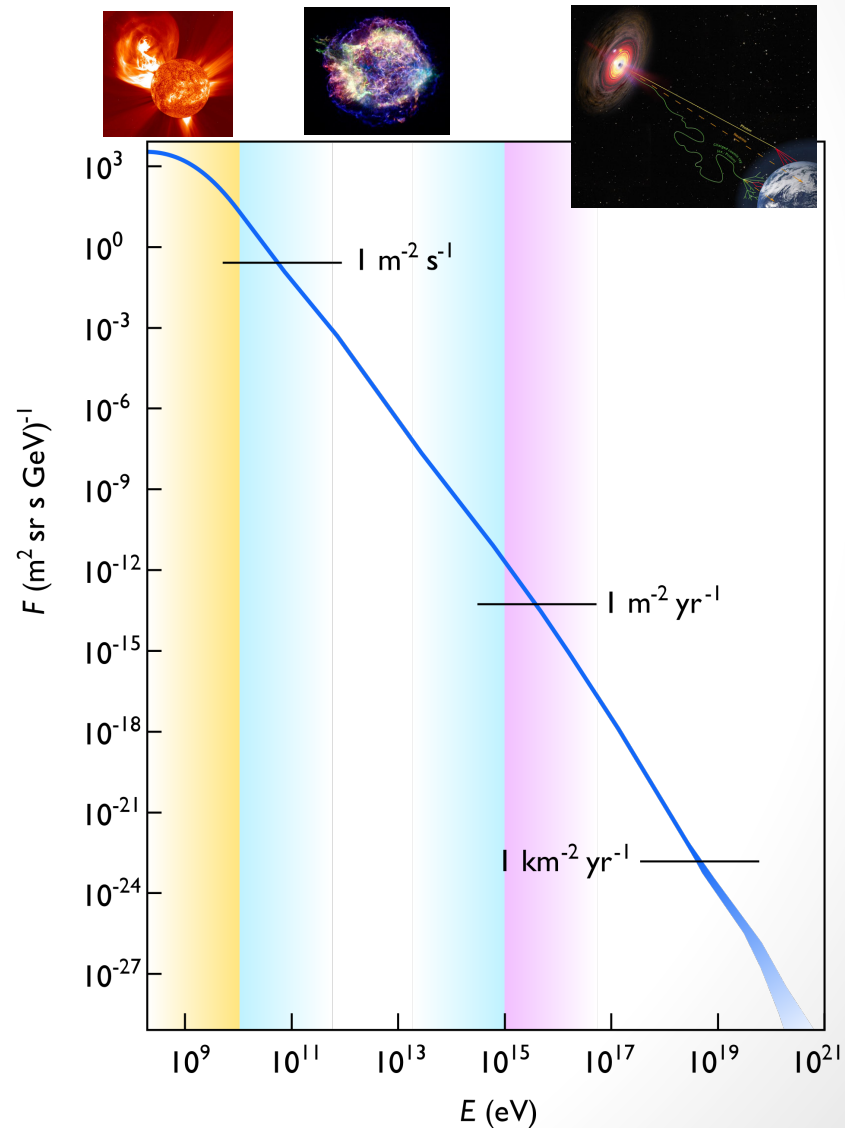


Context

How do we know there is particle acceleration? 3 Examples

1. Cosmic Rays

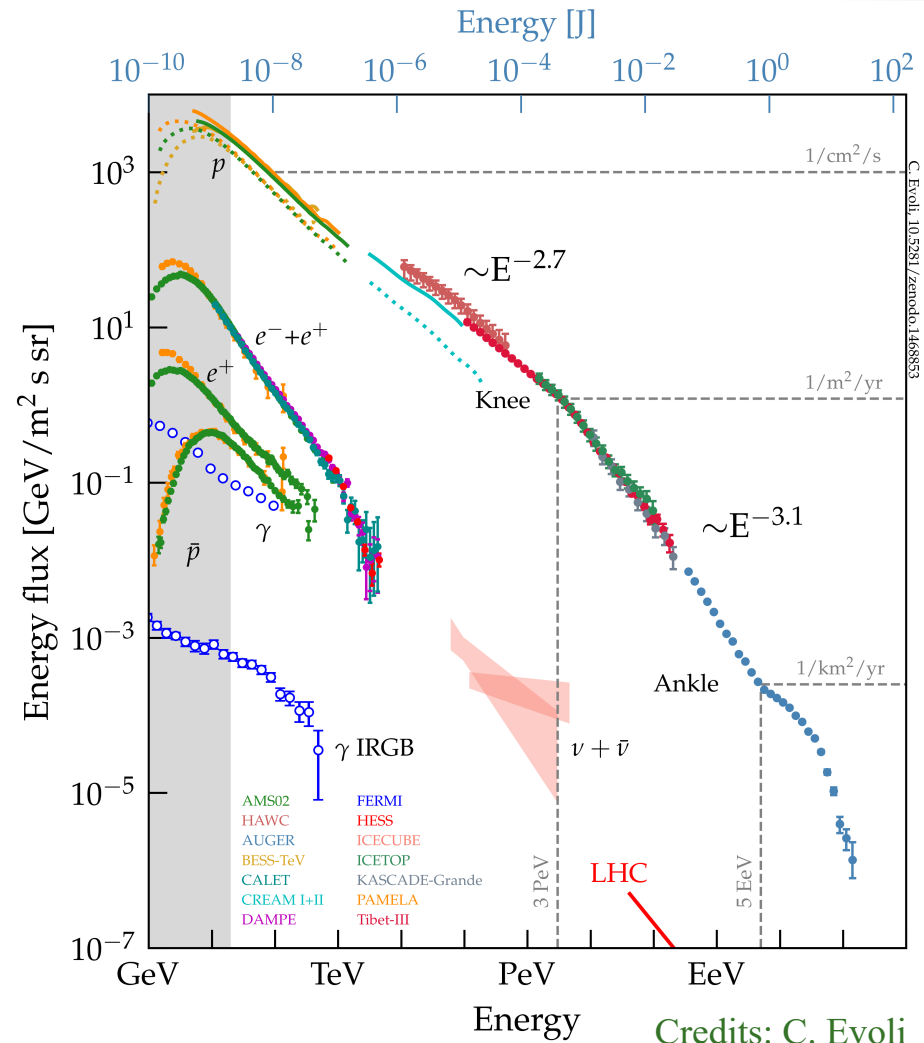
- Charged particle flux measured on ground and in space by dozens of observatories.
- One of most impressive power-laws in astrophysics.
« (second) Great Power Law In The Sky ». First one is for turbulence.
- Fantastic energies, above 10^{20} eV.
(> kinetic energy of a tennis ball launched at 100 km/h)

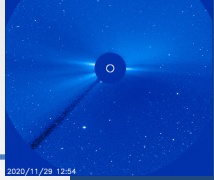


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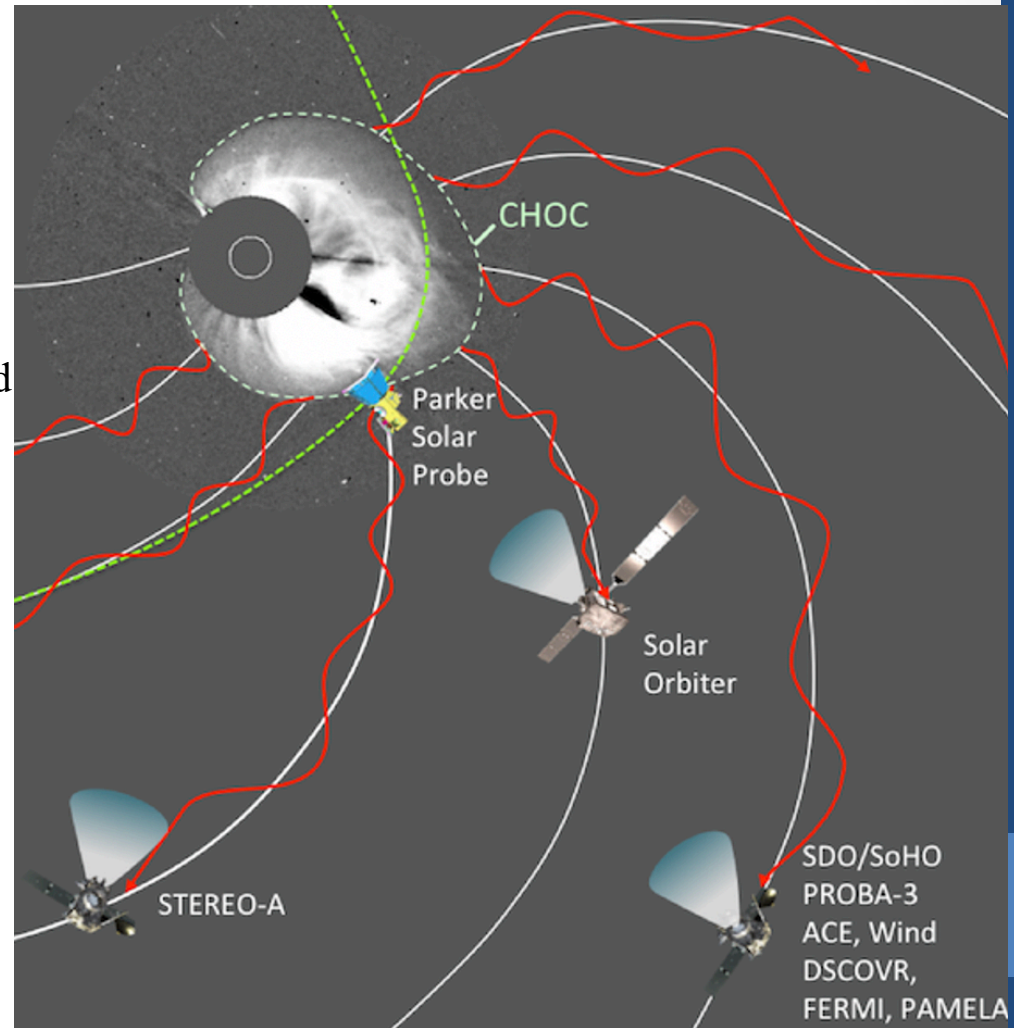


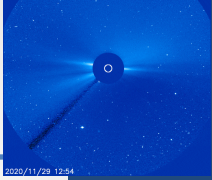


How do we know there is particle acceleration? 3 Examples

2. In-Situ s/c measures (ex: flare / CME of 29-Nov-2020)

- Charged particle flux measured **in-situ** (in space). Current multi-point Helio-Observatory enriched regularly with new s/c (e.g., PSP, SolO).



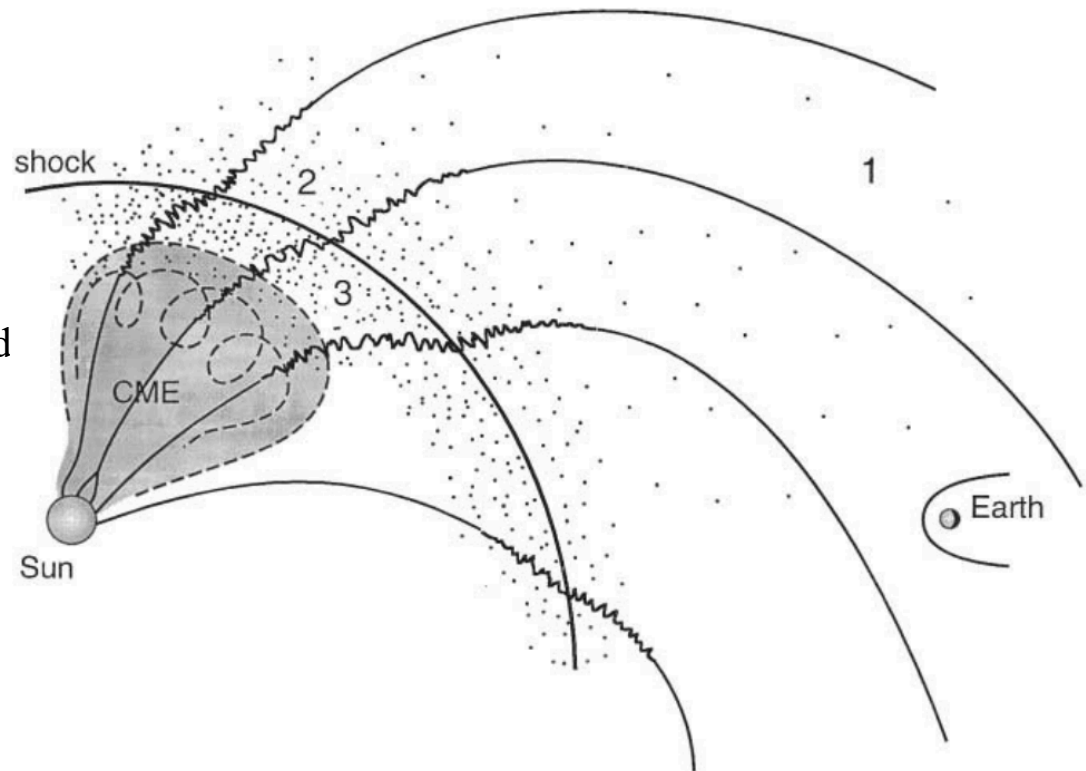


How do we know there is particle acceleration? 3 Examples

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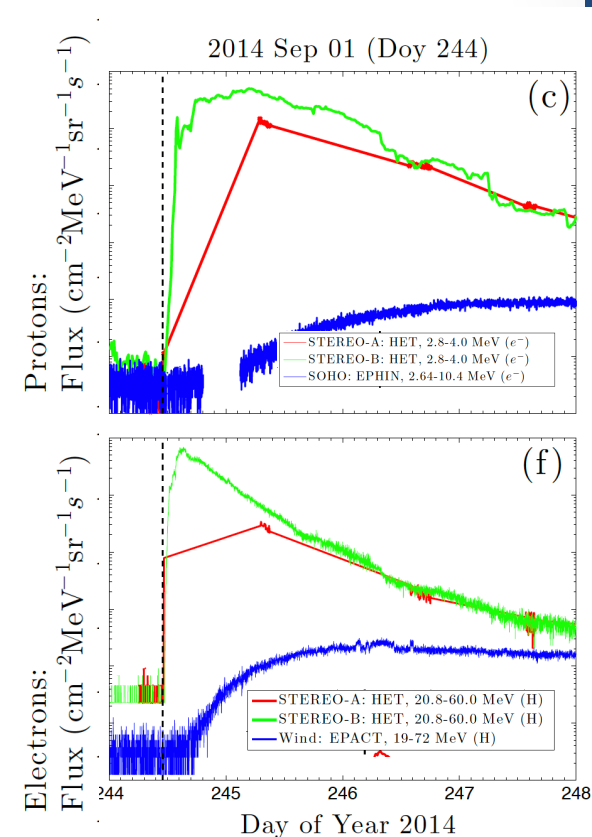


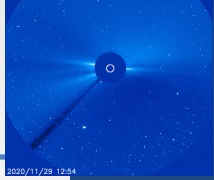
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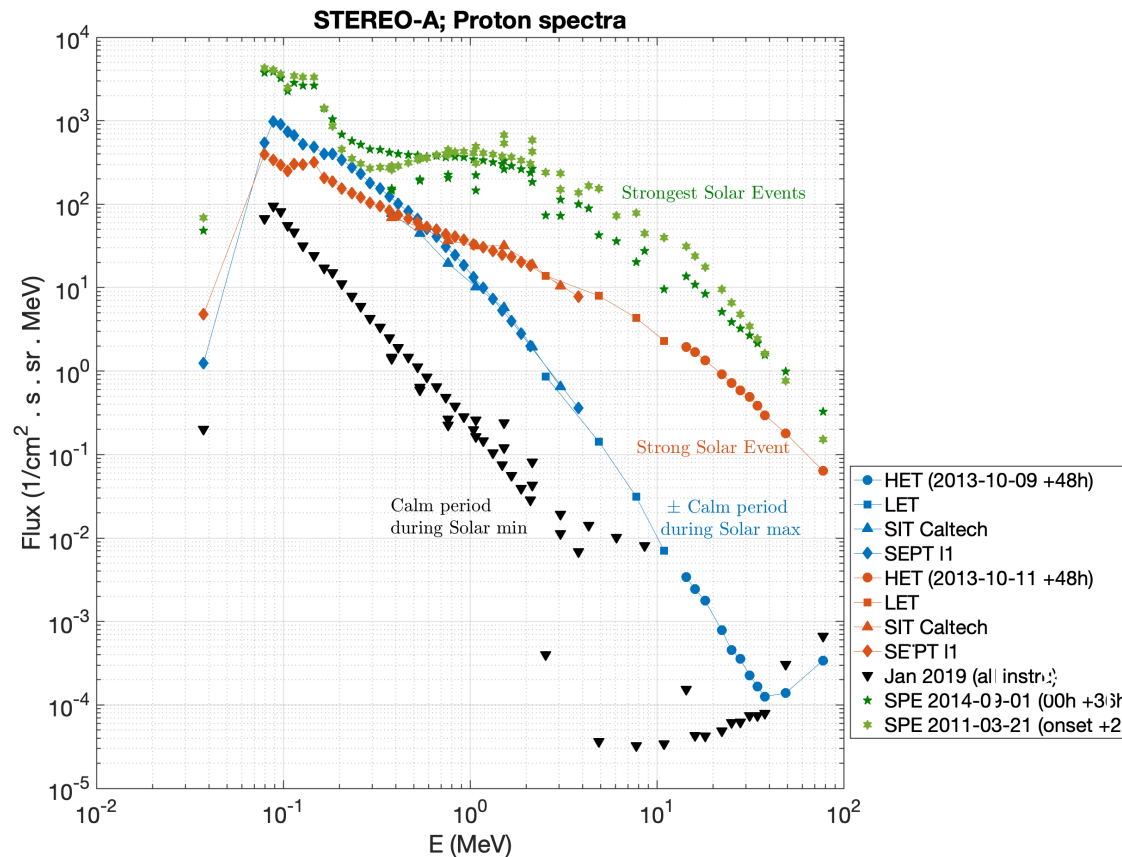


How do we know there is particle acceleration? 3 Examples

2. In-Situ s/c measures

(ex: flare / CME of 29-Nov-2020)

- Charged particle flux measured **in-situ** (in space). Current multi-point Helio-Observatory enriched regularly with new s/c (e.g., PSP, SolO).
- Flux in \gg MeV protons can increase by more than 5 folds in minutes (events are very dynamic).
- Spectra are **non-thermal**
- Energies up to several GeV at extreme solar eruptions (e.g. 10-Sep-2017)

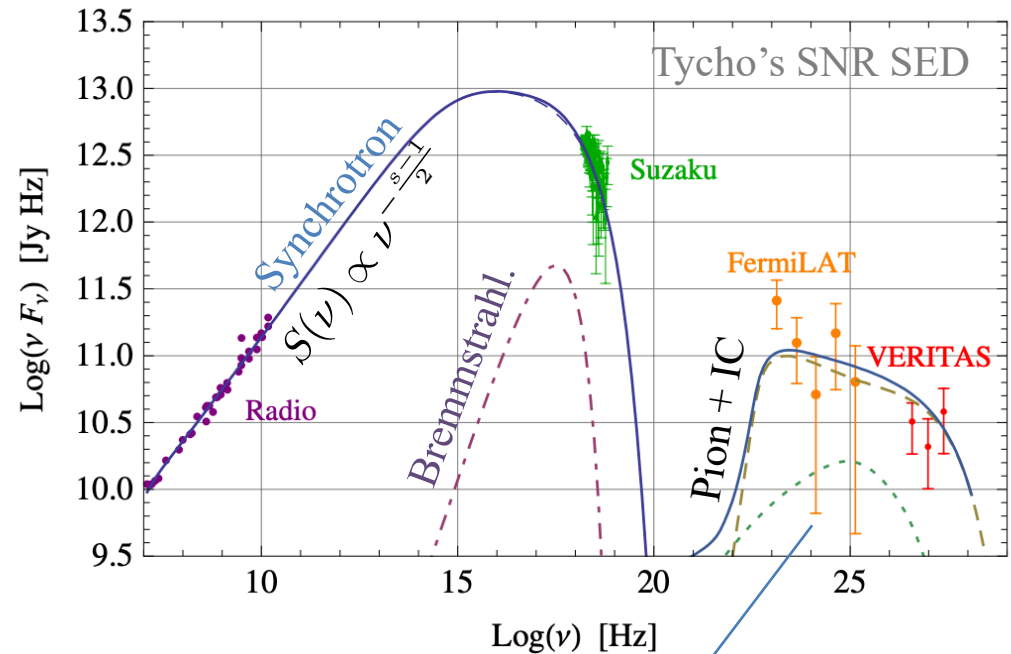


How do we know there is particle acceleration? 3 Examples

3. Indirect: non-thermal emission

- Most common approach for astrophysicists.
- From synchrotron to Inverse Compton (or pion emission): need power-law particle distributions.
- Photon energies can reach $> \text{TeV}$ (means the presence of PeV protons)
- Ignorance of micro-physical parameters typically parametrized

Morlino & Caprioli, A&A, 2012



Implies, for protons, $f(E) = K \cdot E^{-2.2}$

Special place of SuperNova Remnants

Original argument by Baade and Zwicky (1934),
still largely invoked to explain the origin of Galactic CRs.

$$\varepsilon_{\text{CR}} = 0.5 \text{eVcm}^{-3}$$

$$V_{\text{conf}} = \pi R^2 h \approx 2 \times 10^{67} \text{cm}^3 \left(\frac{R}{25 \text{kpc}} \right)^2 \left(\frac{h}{400 \text{pc}} \right)$$

$$W_{\text{CR}} = \varepsilon_{\text{CR}} V_{\text{conf}} \approx 2 \times 10^{55} \text{erg}$$

$$L_{\text{CR}} = \frac{W_{\text{CR}}}{\tau_{\text{conf}}} \approx 5 \times 10^{40} \text{erg.s}^{-1} \left(\frac{\tau_{\text{conf}}}{12 \text{Myr}} \right)^{-1}$$

$$L_{\text{SN}} = f_{\text{SN}} E_{\text{kin}} \approx 6 \times 10^{41} \text{erg.s}^{-1} \left(\frac{f_{\text{SN}}}{0.02 \text{yr}^{-1}} \right) \left(\frac{E_{\text{kin}}}{10^{51} \text{erg}} \right)$$



Messier 1 (T120 at OHP on 2010)



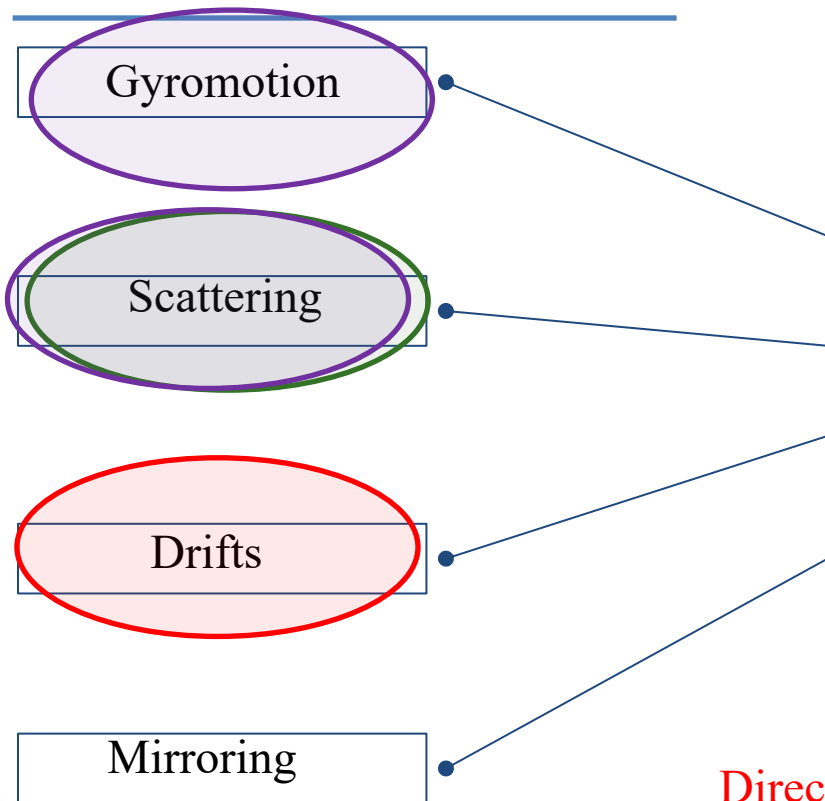
10 % of SN kinetic energy converted into CRs can account for energetics.
→ Originally empirical number. Today motivated by, e.g., SNR and solar shocks observations and kinetic plasma simulations.

Acceleration mechanisms

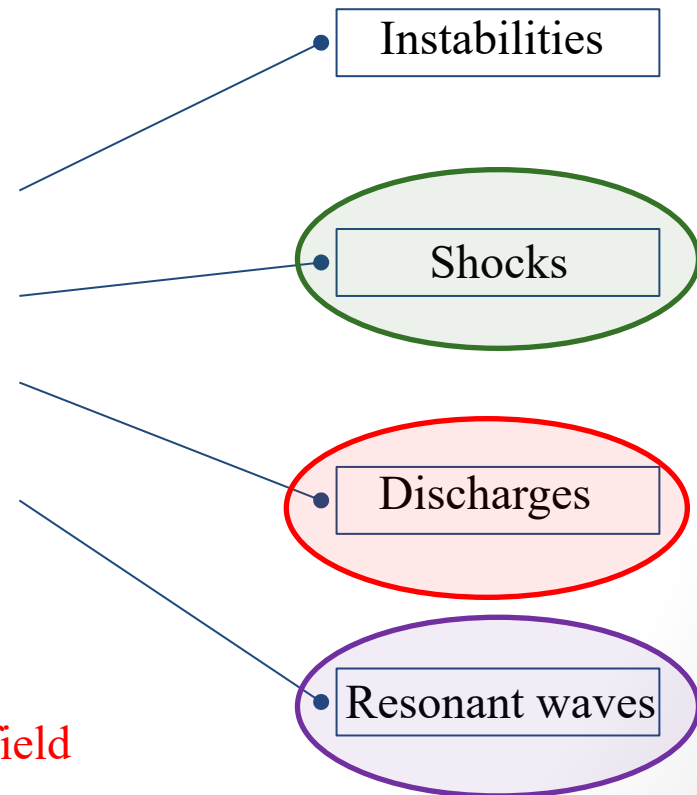
Acceleration Mechanisms

Field-particle interactions

Test-particle kinematics:



Collective plasma effects:



Direct E-field

Fermi II (stochastic)

Fermi I (DSA)

Relevant acceleration mechanisms

Review by D. Melrose 1996

2. Possible acceleration mechanisms

Acceleration mechanisms that are thought to be important for fast particles in astrophysical and space plasmas include the following:

1) Stochastic acceleration by MHD waves

2) Diffusive shock acceleration

3) Resonant acceleration by MHD waves

4) Resonant acceleration by Langmuir waves

5) Shock drift acceleration

6) Acceleration during magnetic reconnection

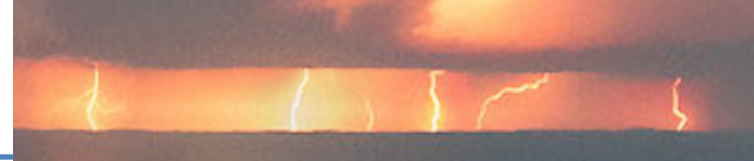
7) Runaway acceleration by a parallel electric field

8) Acceleration by potential double layers

Fermi acceleration II and I

$$\omega - s\Omega - k_{\parallel}v_{\parallel} = 0$$

Lots of recent progress
(but not discussed here)



Direct acceleration by electric field



$$\frac{d\vec{p}}{dt} = q \left(\vec{E} + \vec{v} \times \vec{B} \right) = 0 \text{ (if static: no work done)}$$

Assume relativistic particle ($\epsilon = pc$) and electric field with a given coherence length L_c .
 The energy gain by particle is then: $\Delta\epsilon = ZecE_{\parallel} \Delta t = ZeE_{\parallel} L_c$

A more general expression (guiding-center approximation):

$$\begin{aligned} \frac{dW}{dt} &= ev_{\parallel} E_{\parallel} + e\dot{\mathbf{R}}_{\perp} \cdot \mathbf{E} + M \partial B / \partial t \\ &= \boxed{ev_{\parallel} E_{\parallel}} + M \mathbf{u}_{\mathbf{E}} \cdot \nabla B + mv_{\parallel} \mathbf{u}_{\mathbf{E}} \cdot \frac{d\hat{e}_{\parallel}}{dt} + m \mathbf{u}_{\mathbf{E}} \cdot \frac{d\mathbf{u}_{\mathbf{E}}}{dt} + M \frac{\partial B}{\partial t} \end{aligned}$$

Northrop (1963)

Grad B
drift

Inertial
drift

Polarization
drift

Induction

- Issue: plasma in space is an extremely good conductor. Electric fields are rapidly screened: coherent E-field is generally not sustained over large distances. Acceleration not efficient to highest energies of Cosmic Rays

Fermi's idea

PHYSICAL REVIEW

VOLUME 75, NUMBER 8

APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI

Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

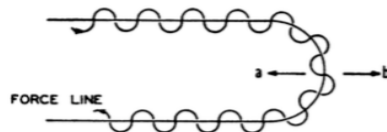


FIG. 1. Type B reflection of a cosmic-ray particle.

the energy. An elementary calculation shows that the probability for a particle to have energy between w and $w+dw$ is given by

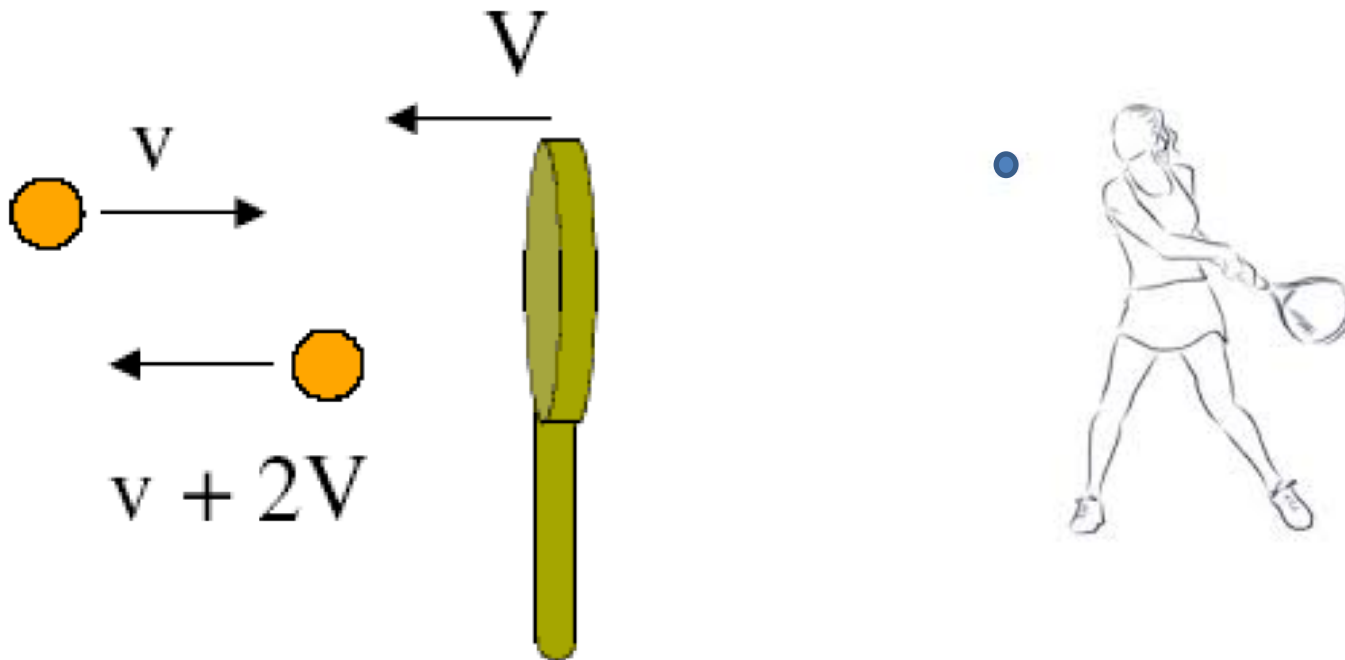
$$\pi(w)dw = (\tau/B^2T)(Mc^2)^{\tau/B^2T}dw/w^{1+\tau/B^2T}. \quad (10)$$

It is gratifying to find that the theory leads naturally to the conclusion that the spectrum of the cosmic radiation obeys an inverse power law. By

Requires: moving magnetic bottles, magnetic walls or curved field lines.
Statistical gain of energy by particles, naturally results in a power-law distribution.

Analogy

- Analogy: tennis ball and player's racket



In the frame of the racket: no energy change. Change by double frame transformation.

In the following: $\beta = V/c \simeq V/v$

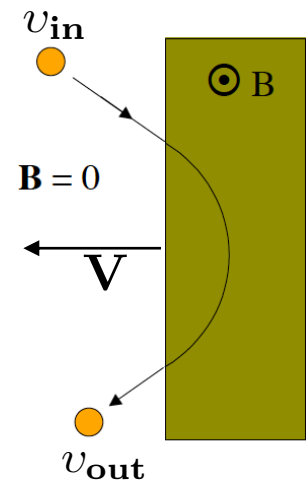
How it works (1): Double Lorentz transform

- Assumption: In the magnetic irregularity (wave) frame the scattering of the particle is elastic.
- Transform to the wave frame \rightarrow scatter \rightarrow transform back to the lab frame

$$\begin{cases} E'_{\text{in}} = \gamma E_{\text{in}}(1 - \beta \cos \theta_{\text{in}}) \\ E_{\text{out}} = \gamma E'_{\text{out}}(1 + \beta \cos \theta'_{\text{out}}) \end{cases} ,$$

$$E_{\text{out}} = \gamma^2 E_{\text{in}}(1 - \beta \cos \theta_{\text{in}})(1 + \beta \cos \theta'_{\text{out}}),$$

$$\frac{\Delta E}{E} = \frac{\beta(\cos \theta'_{\text{out}} - \cos \theta_{\text{in}}) + \beta^2(1 - \cos \theta_{\text{in}} \cos \theta'_{\text{out}})}{1 - \beta^2}.$$



- Meaning: kinetic energy of the moving magnetic cloud is transmitted to the particle (if frontal) or the opposite if catching-up

How it works (2) : Scattering statistics

$\langle \cos \theta_{\text{out}} \rangle = 0$ isotropic escape from magnetic cloud

$$\langle \cos \theta_{\text{in}} \rangle = \frac{\int_{-1}^1 \cos \theta_{\text{in}} (v - V \cos \theta_{\text{in}}) d(\cos \theta_{\text{in}})}{\int_{-1}^1 (v - V \cos \theta_{\text{in}}) d(\cos \theta_{\text{in}})} = \frac{-2V/3}{2v} \simeq -\frac{1}{3}\beta,$$

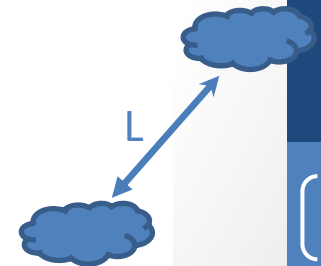
The average energy gain is then:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3} \frac{\beta^2}{1 - \beta^2} \simeq \frac{4}{3} \beta^2.$$

Original second-order acceleration, in V/c .

Acceleration time : $\tau_{\text{acc}} = \frac{\Delta t}{\langle \Delta E/E \rangle} = \frac{2L/c}{4/3(V/c)^2} = \frac{3Lc}{2V^2}$

For relativistic particles, does not depend on E.



How it works (3) : Power-Law formation

In general terms we require:

- Stationary injection (at E_0). *Assumed*
- Acceleration rate independent of E . *Verified*
- Escape probability independent of E . *Assumed*

Then: $E(t) = E_0 \exp(t/\tau_{\text{acc}}) \rightarrow t(E) = \tau_{\text{acc}} \log(E/E_0)$

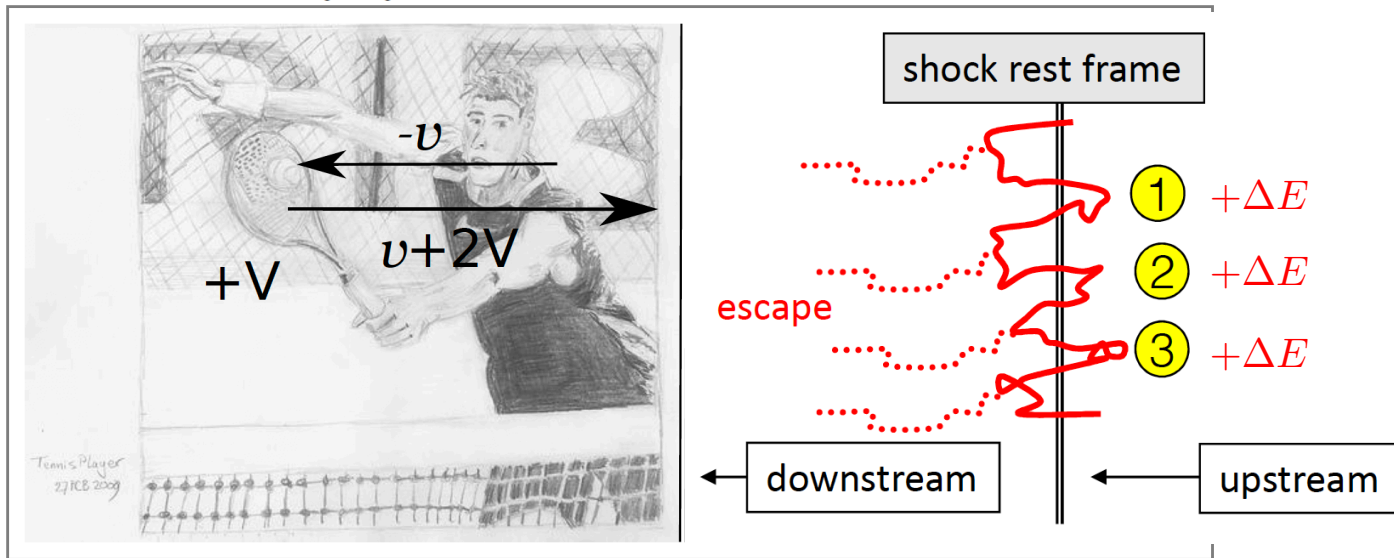
$$\text{And } N(E) = \dot{N}_0 \tau_{\text{acc}} \exp(-t(E)/\tau_{\text{esc}})$$
$$f(E) = \frac{dN}{dE} \propto \left(\frac{E}{E_0}\right)^{-(1+\tau_{\text{acc}}/\tau_{\text{esc}})}$$

Resulting in a power-law distribution.

But, the typical average speed of magnetic clouds is too small to explain CRs. I.e., acceleration time (10^8 yr) $>$ residence time in the Galaxy (10^7 yr). Scattering on wave (magnetic turbulence) favored since first Fermi works.

Fermi acceleration at shocks: 1st order (or DSA)

Shocks: convergent flows with frozen-in magnetic turbulence.
Gain by cycles on both sides of the shock.



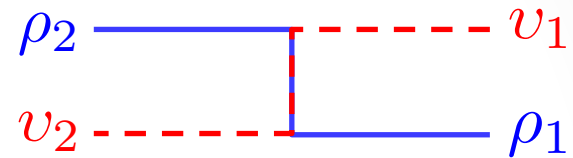
Credits: M. Lemoine

As the flow pattern is convergent: frontal 'collision' -> Energy gain is systematic

Fermi acceleration at shocks: A bit of shock physics

Shock is a well-defined transition.

Conservation laws in shock front rest frame
(mass, momentum and energy):



$$\begin{aligned} \rho_2 v_2 &= \rho_1 v_1 \\ p_2 + \rho_2 v_2^2 &= p_1 + \rho_1 v_1^2 \\ \rho_2 v_2 (v_2^2/2 + p_2/\rho_2 + e_2) &= \rho_1 v_1 (v_1^2/2 + p_1/\rho_1 + e_1), \end{aligned}$$

} Modified if relativistic
shock motion
(AGNs, GRBs, ...)

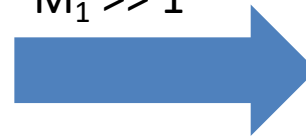
After some algebra, and using the standrad EoS:

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{\gamma + 1}{\gamma - 1 + 2M_1^{-2}}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma - 1)][\gamma - 1 + 2M_1^{-2}]}{(\gamma + 1)^2}$$

$M_1 \gg 1$



$$r = \frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = 4$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{2\gamma M_1^2 (\gamma - 1)}{(\gamma + 1)^2}$$

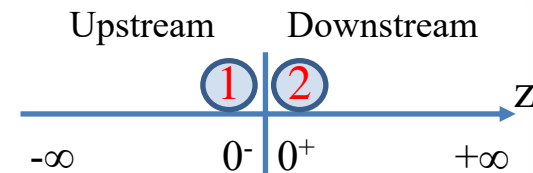
Fermi acceleration at shocks: 1st order (or DSA)

Kinetic approach (Krymskii 1977, Blandford & Ostriker 1978, etc):

Parker equation:
$$u \frac{\partial f}{\partial z} = \frac{\partial}{\partial z} \left[D \frac{\partial f}{\partial z} \right] + \frac{1}{3} \frac{du}{dz} p \frac{\partial f}{\partial p} + Q$$

Advection Diffusion Ad Change Injection

At the shock front:
$$\frac{dv}{dz} = (v_1 - v_2) \delta(z)$$



Integrate in different zones and join at the front. Leads to:

$$\rightarrow \frac{df}{f} = \frac{3v_1}{v_2 - v_1} \frac{dp}{p} \rightarrow f \propto p^{-\frac{3v_1}{v_1 - v_2}}$$

Power-law index depends only on the compression ratio, as $v_1/v_2 = r$! No dependence on diffusion coefficient.

... But the acceleration rate does:

$$\tau_{\text{acc}} = \frac{3}{v_1 - v_2} \left(\frac{D_1}{v_1} + \frac{D_2}{v_2} \right) \longrightarrow \sim 1 \text{ month in SNRs!}$$

Fermi acceleration at shocks: 1st order (or DSA)

Intuitive approach (Bell 1978):

Scattering Statistics: $\langle \cos \theta_{\text{in}} \rangle = \frac{\int_{\pi}^{\pi/2} \cos \theta_{\text{in}}^2 \sin \theta_{\text{in}} d\theta_{\text{in}}}{\int_{\pi}^{\pi/2} \cos \theta_{\text{in}} \sin \theta_{\text{in}} d\theta_{\text{in}}} = -\frac{2}{3},$

Energy gain per cycle: $\langle \Delta E \rangle = \frac{4}{3} \beta E = \frac{4}{3} \frac{\Delta v}{c} E.$ **First order**

Escape probability: $\mathcal{P}_{\text{esc}} = \frac{\Phi_{\text{esc}}}{\Phi_{\text{ud}}} = \frac{4v_2}{v} = \frac{4}{r} \frac{V_{\text{choc}}}{v} = \frac{4}{r} \beta_{\text{choc}},$

Formation of a power law

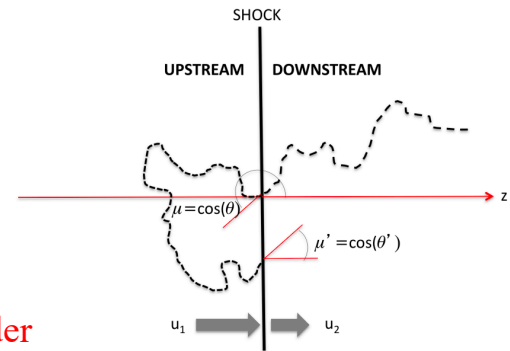
$$E_n = (1 + k)^n E_0 ; \quad k = \frac{4}{3} \frac{r - 1}{r} \beta_{\text{choc}}.$$

$$N_n = N_0 \mathcal{P}_{\text{ret}}^n = N_0 (1 - \mathcal{P}_{\text{esc}})^n$$

$$n = \frac{\log(E/E_0)}{\log(1+k)} \implies N(\geq E) = N_0 (1 - \mathcal{P}_{\text{esc}})^{\frac{\log(E/E_0)}{\log(1+k)}},$$

Power-law index

$$N(E) = (x - 1) \frac{N_0}{E_0} \left(\frac{E}{E_0} \right)^{-x} ; \quad x = \frac{r+2}{r-1}.$$



Fermi acceleration at shocks: 1st order (or DSA)

Strong points of DSA at SNR shocks

- SNe supply enough energy to account for CR luminosity. Need to extract 10% of SNR kinetic energy into CRs.
- Acceleration at shocks is observed in-situ in the Heliosphere: ICMEs, CIRs, planetary bow-shocks.
- Power-law index is universal, in test-particle limit. For strong shocks with Mach $\gg 1$, $s=2$. Close to one measured for CRs.
- The acceleration time is generally shorter than others.

... But important difficulties:

- Inferred spectrum from young SNRs does not match perfectly. It is closer to $s = 2.3$. Need for more complete theories.
- Microphysics are not well in-hand (dependence on composition, upstream B-field inclination)... but lots of recent progress.

Beyond standard DSA: non-linearities

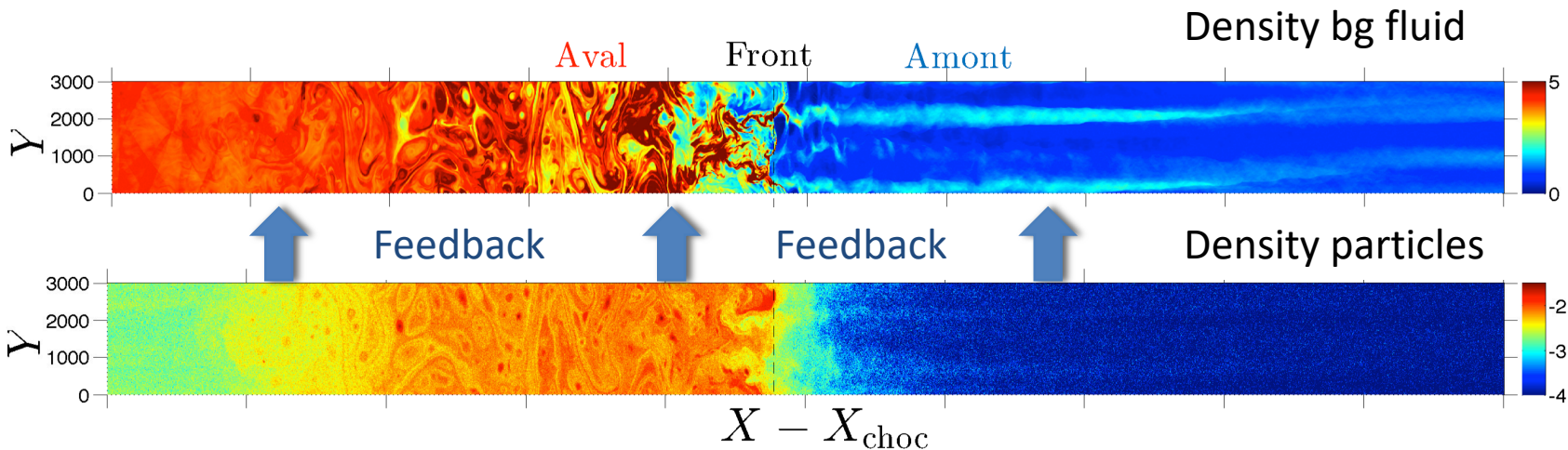
Problematic 1: 10% of total energy going into accelerated particles is well beyond the test-particle limit.

Problematic 2: inferred power-law index for protons in young SNRs is steeper than $s = -2$, expected universally in standard DSA.

Non-linear kinetic eqs firstly solved by [Blasi \(2002\)](#) in SNR context (but successful quasi-linear solutions presented by [Martin Lee \(1982, 83\)](#) in heliospheric context).

Most of recent work focuses on this approach.

Beyond standard DSA: non-linearities



In no strong feedback \rightarrow test-particle DSA is valid.

If feedback from accelerated particles (non-linearity): changes the shock structure, magnetic field amplification (e.g., Bell instability) and acceleration properties (acc. time, slope).

Some references and useful links

- Fermi (1949), Physical review, 75, 1169
 - Fermi (1954), Astrophys.J.119, 1
 - Krymskii (1977), Dokl. Akad. Nauk SSSR 234, 1306
 - Bell (1978), MNRAS, 182, 147
 - Blandford & Ostriker (1978),
 - Axford et al. (1977), Proc. 15th Int. CR Conf.,11, 132
 - Lee, Martin; (1983), JGR, 88, A8, 6109
 - Blandford & Eichler (1987), Phys. Rept., 154
 - Achterberg (2003), Les Houches, vol. 78, p.313
 - Lemoine M., (2019), PRD, 99,083006
 - Caprioli (2012), JCAP, 07, 038
- } Foundation articles
- } DSA classical papers
- } Good reviews on DSA
- } Generalized Fermi acceler.
- } Non-linear DSA

Some useful links (in French):

- Cours d'Etienne Parisot a l'ecole de Goutelas:
http://www.apc.univ-paris7.fr/~parizot/documents/parizot_CoursGoutelas2003.pdf
- Cours de Guy Pelletier, 2007, OHP, Ecole Astroparticules:
http://www.cpt.univ-mrs.fr/~cosmo/EcoleAstroparticules/Cours_EAP1.html

Backup

Hillas criterion: simple argument for E_{\max}

$$r_g = \frac{E}{Z_{\text{eff}} e B_0} \longrightarrow \text{When } r_g(E) = R_{\text{source}} \longrightarrow E_{\max} = ZeB_0 R_{\text{source}}$$

