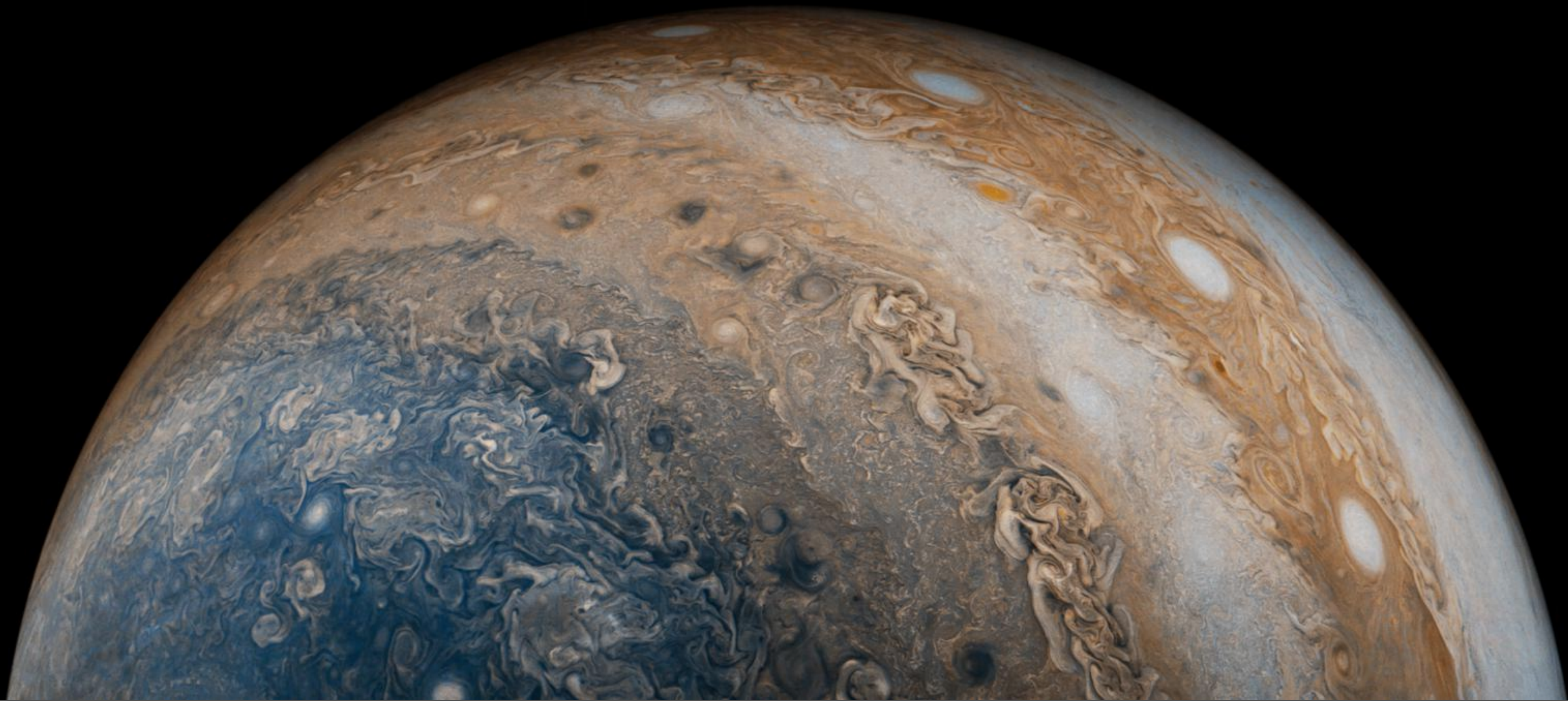
AN INTRODUCTION TO DENSE PLASMAS IN GIANT PLANETS AND BROWN DWARVES

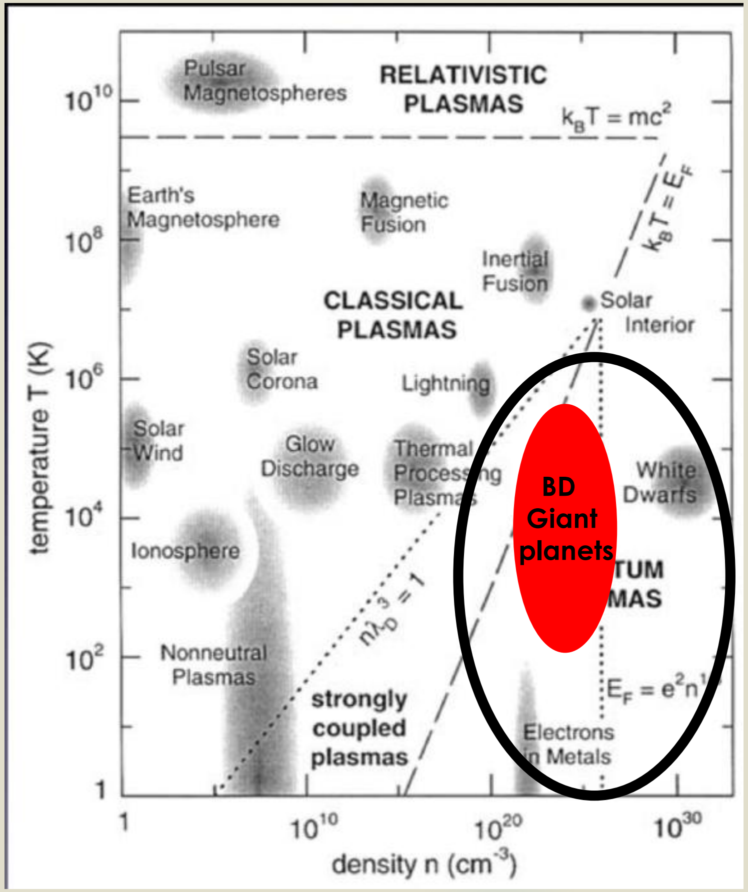


Astroplasma

First session in dense plasmas

Introduction today
In two weeks: experiments and simulations

Looking for someone on the white dwarf/neutron star thematics



Overview

I. *Overview of giant planet interior*

II. *Metallic hydrogen*

III. *H/He immiscibility*

I. Overview of giant planet interior

	Jupiter	Saturn	HN Peg B
Mass	317 $\mathrm{M_T}$	95 $\mathrm{M_T}$	8876 $\mathrm{M_T}$
Radius	70 000 km	58 000 km	~ 80 000 km
Mean density	1300 $\mathrm{kg.m^{-3}}$	700 $\mathrm{kg.m^{-3}}$	20 000 $\mathrm{kg.m^{-3}}$
Pressure ($\frac{GM^2}{\frac{4}{3}\pi R^4}$)	$210^{12}\mathrm{Pa}$	$210^{11}\mathrm{Pa}$	$10^{15}\mathrm{Pa}$

I. Overview of giant planet interior

Beginning of the 20th century: assumed to be cold bodies. No T ionisation

Only possibility: mostly solid hydrogen.

But: $\varrho \approx \frac{m_H}{\frac{4}{3}\pi d^3}$ hence $d \approx 610^{-11}\, m$ for Jupiter, $8\, 10^{-11}\, m$ for Saturn.

Mean distance ≈ Bohr radius: electron orbit overlap

I. Overview of giant planet interior

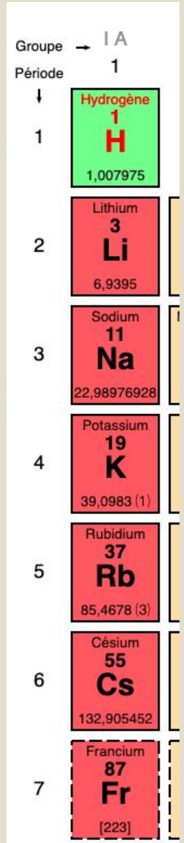
Delocalisation of electrons: conductivity increases drastically

Hydrogen deserves to be called “alkali metal”

Degenerated electron: degeneresence pressure
(Fermi exclusion)

Difficulty: interactions >> kinetics. No analytical equation of state

The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble. Dirac, 1929



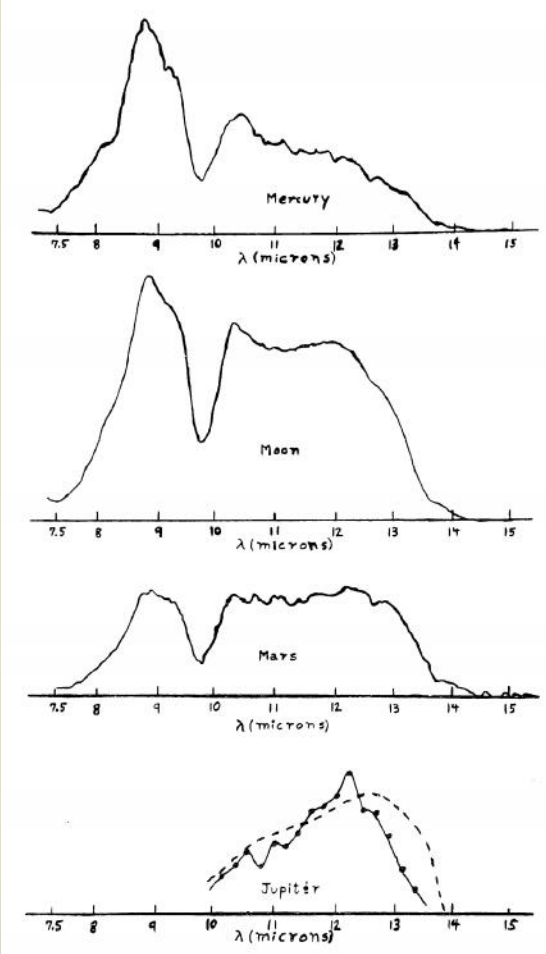
I. Overview of giant planet interior

1950-1970

Demarcus 1958: Jupiter & Saturn ~ solar composition

Low 1964: determination of the infrared flux of Jupiter

Internal flux = 1.7 times solar flux !



I. Overview of giant planet interior

1950-1970

Peebles 1964, Hubbard 1968: hot, convective, liquid H
("first use of electronic computer")

Coherent with presence of magnetic field (~1960s)

Hubbard 1968: internal temperature
~ 20 000K < H2 binding << H ionisation

Transition from H2 to metallic H ?

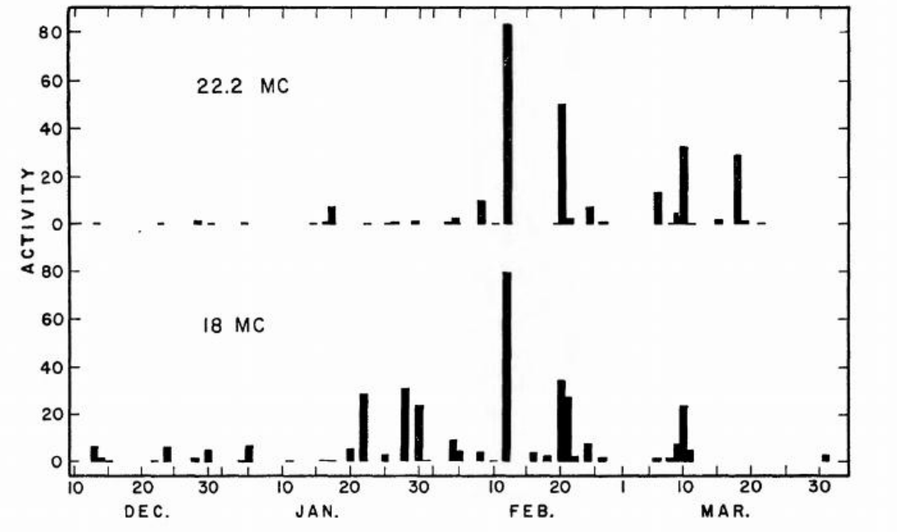


FIG. 1.—Jovian activity recorded at the University of Florida during the 1957–1958 apparition

I. Overview of giant planet interior

Evolution

Evolution: convective cooling + radiative loss at the photosphere

Age of Jupiter ~ 4.5 Gyr. Age of Saturn ~ 2 Gyr. (Everyone in early 1970s)

What is the difference between Saturn and Jupiter ?

I. Overview of giant planet interior

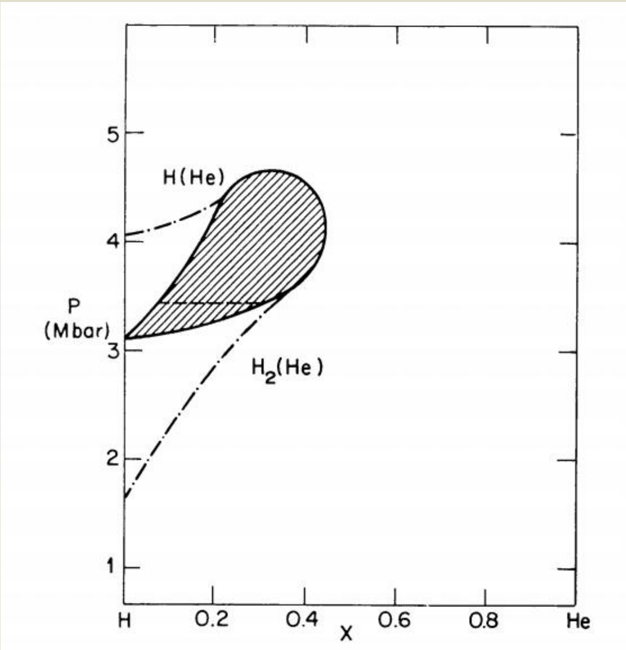
Smoluchowski 1967, Salpeter 1973, Stevenson & Salpeter 1977a, b

Neutral helium is immiscible in metallic H at low temperature and pressure.

Cooling ➔ gravitational layering ➔ energy source

I. Overview of giant planet interior

Cooling ➔ gravitational layering ➔ energy source



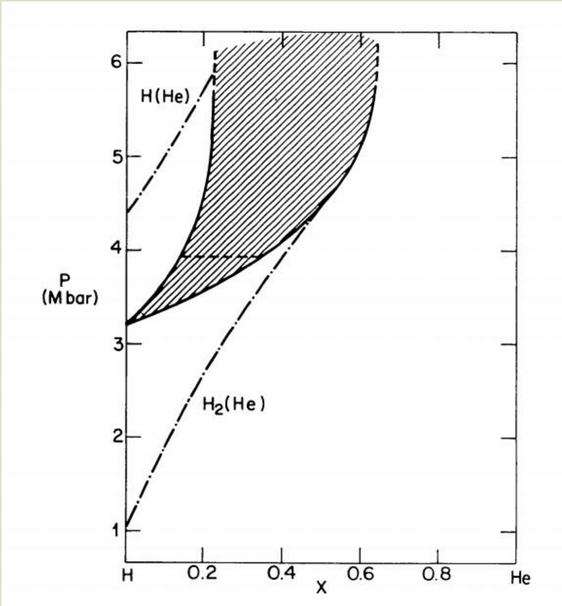
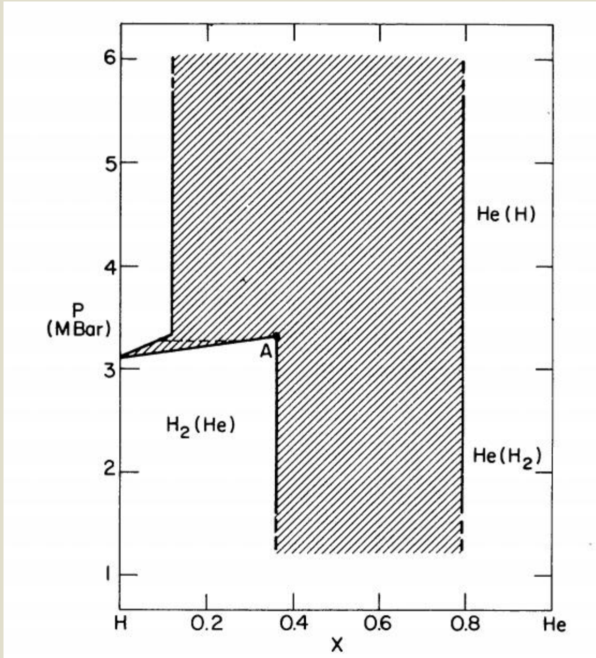
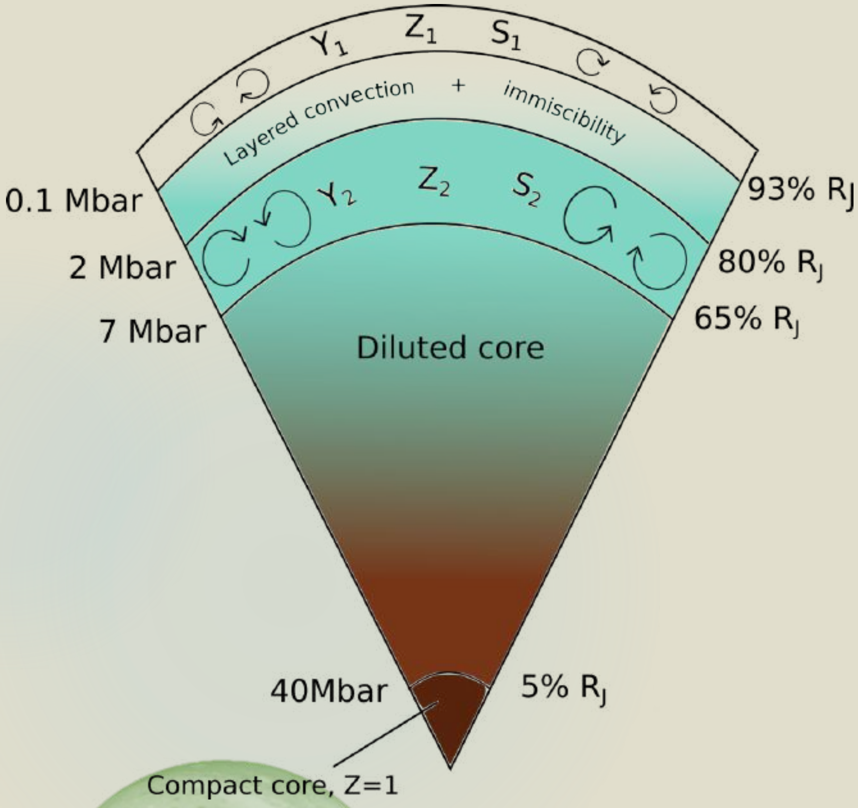


FIG. 5b



I. Overview of giant planet interior



Debras & Chabrier 2019

Juno (2016) + Galileo (1995)
Latest EOS (2019) + H-He immiscibility (2013-2020)

Non completely convective planet

Drastically relies on the understanding of H and He behaviour at high pressure

Will change with experimental physics and simulations

II. Metallic hydrogen

Metallisation: Wigner & Huntington 1935, Kothari 1938, Wild 1938

EOS: Chabrier et al. 1992, Saumon et al. 1995, Nellis 2000, Chabrier et al. 2020, Chabrier & Debras 2021

Experiments: Nellis et al. 1995 , Loubeyre papers, Celliers et al. 2018

Simulations: Mazzola et al. 2018, Morales et al. 2013, Lorenzen et al. 2009

II. Metallic hydrogen

We will follow Kothari 1938 methodology

Assumption: the kinetic energy only comes from electrons

Cold material: bound and free electrons degenerated

Verification: $T_F \approx \frac{\hbar^2}{2m_e k_B}(3\pi^2 n)^{\frac{2}{3}} \approx 7\ 10^5 K \text{ Jupiter}, 4\ 10^5 K \text{ Saturn}$

II. Metallic hydrogen

Goal: obtaining analytically the degree of ionization

Virial theorem: $2E_{kin} + W = 3pV$

From Fermi energy, we get total kinetic energy

$$E_{kin} = \frac{3}{5}\frac{N{p_F}^2}{2m} \approx \frac{\rho V}{m_H}\frac{3\hbar^2}{10m_e}\left(\frac{3\pi^2\rho}{m_H}\right)^{2/3}$$

II. Metallic hydrogen

Potential energy ? Simplification: sum of electrostatic energy of individual cells containing one proton and one electron (bound or free) uniformly distributed in the cell.

$$U(x) \approx \frac{1}{4\pi\epsilon_0 x}\left(e - \frac{4}{3}\pi n^* x^3 e\right) = \frac{1}{4\pi\epsilon_0 x}(e - \frac{x^3}{a^3}e)$$

$$W' = \int_0^a \frac{1}{4\pi\epsilon_0 x}\left(e - \frac{x^3}{a^3}e\right) n^*(-e)\, 4\pi x^2 dx = -\frac{9}{40\pi\epsilon_0}\frac{e^2}{a}$$

II. Metallic hydrogen

Total potential energy is the sum:

$$W = -\frac{\rho V}{m_H}\frac{9}{40\pi\epsilon_0}e^2\left(\frac{\rho}{m_H}\right)^{1/3}$$

In our case, external pressure comes from hydrostasy, and must be equal to degenerate pressure:

$$p = \frac{8\pi}{15}\frac{h^2}{m_e}\left(\frac{3n}{8\pi}\right)^{5/3}$$

n is the FREE electron concentration, contrary to n* the total electron concentration

II. Metallic hydrogen

Denoting μ the mean molecular weight per FREE electron:

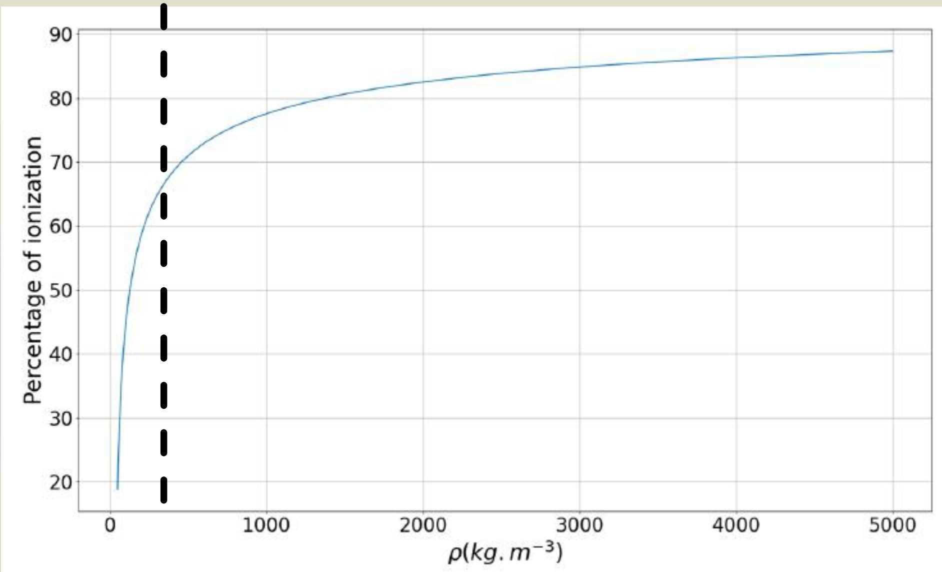
$$n = \frac{\rho}{\mu m_H}$$

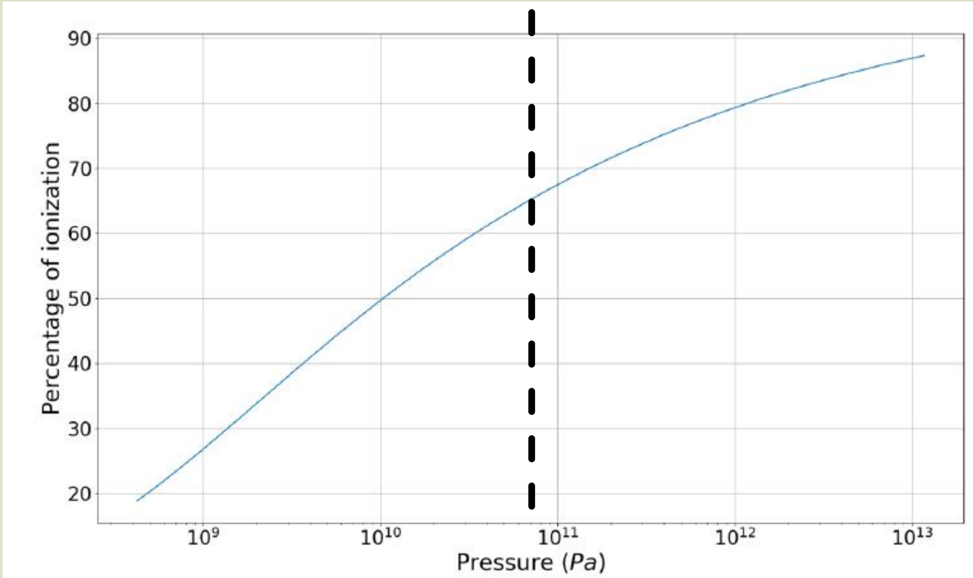
We have: $p = \frac{K\rho^{5/3}}{\mu^{5/3}}$ and from the Virial theorem:

$$\mu \approx \frac{1}{\left[1 - \left(\frac{\Delta}{\varrho}\right)^{1/3}\right]^{3/5}}, \quad \Delta = \left[\frac{e^2 m}{\varepsilon_0 h^2}\left(\frac{m_H}{8\pi}\right)^{1/3}\right]^3$$

II. Metallic hydrogen

Important to notice: the degree of ionization only depends on p or ρ.





Dashed line: 90% of Jupiter, 50% of Saturn

II. Metallic hydrogen

Compares pretty well with other (zero temperature) EOS.

Effect of temperature: ionisation much more efficient with pressure, but complete loss of analytics.

Interesting application: maximum radius of planets and brown dwarves (and maximum mass of white dwarves)

II. Metallic hydrogen

Taking mean density instead of density:

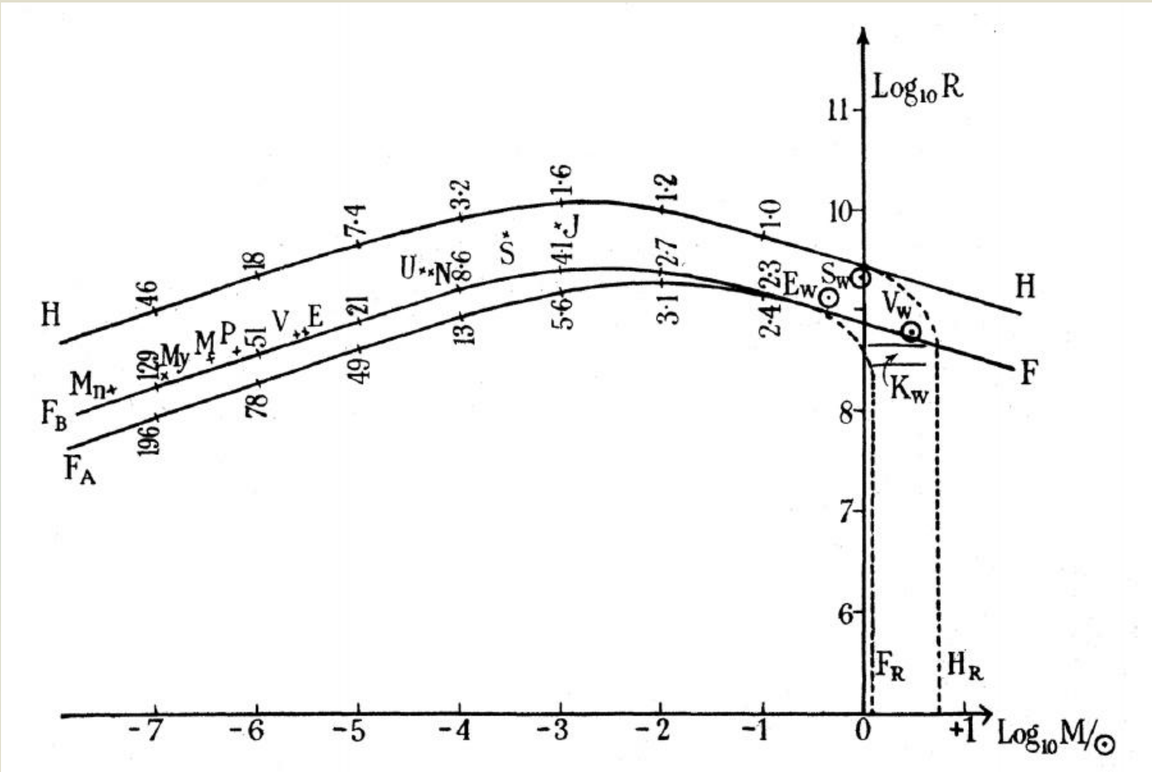
$$\bar{\mu} \approx \frac{1}{\left[1 - \left(\frac{\Delta}{3M/4\pi R^3}\right)^{1/3}\right]^{3/5}}$$

Resolving the virial theorem with gravity $(2E_{kin} + W + W_G = 0)$

$$R = \frac{l\left(\frac{M_\odot}{M}\right)^{1/3}}{1 + l\left(\frac{4\pi\Delta}{3M_\odot}\right)^{1/3}\left(\frac{M_\odot}{M}\right)^{2/3}}$$

$l \approx 3\ 10^7 m$

II. Metallic hydrogen



Result:

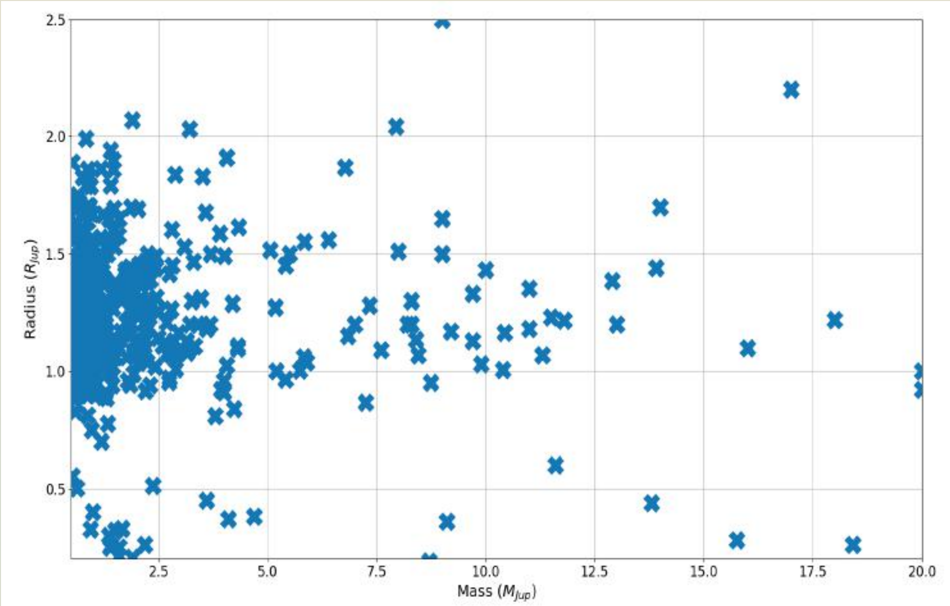
Degenerate matter reproduces well planet M-R relations

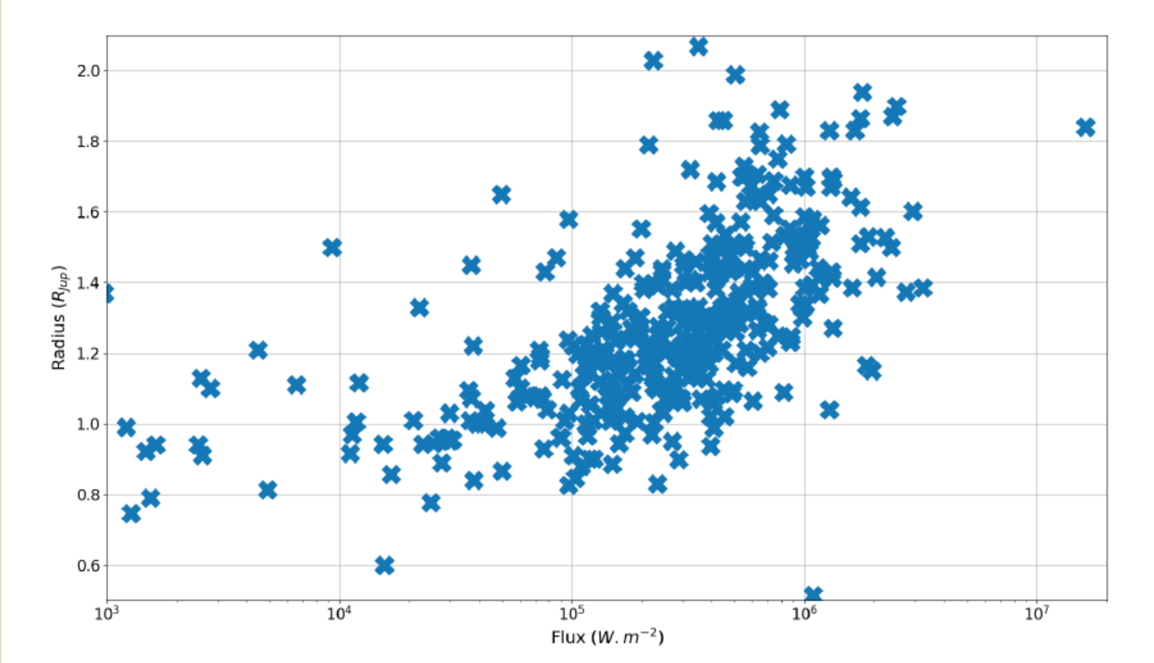
Prediction:

No planet should be bigger in radius than Jupiter

II. Metallic hydrogen

Exoplanets





Very good prediction from Kothari !

II. Metallic hydrogen

How to make an EOS for metallic H ?

SCvH (Saumon et al. 1995), used until 2020.

Based on the chemical picture: minimisation of free energy F. Assuming that atoms and molecules remain definite and using statistical mechanics, not true notably when partial ionization

$$P = -\left.\frac{\partial F}{\partial V}\right|_{T,N}, S = -\left.\frac{\partial F}{\partial T}\right|_{V,N}$$

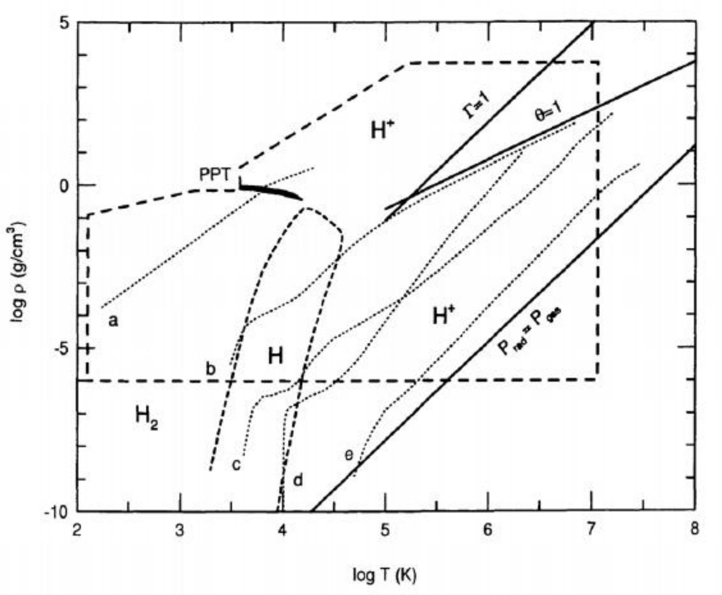
II. Metallic hydrogen

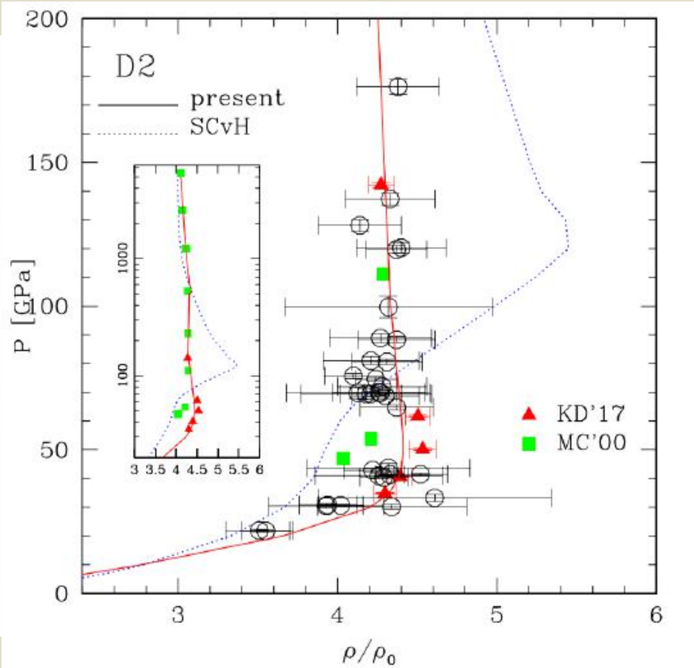
The SCvH EOS solves semi-analytically the minimisation of free energy, and interpolates in region with no solution.

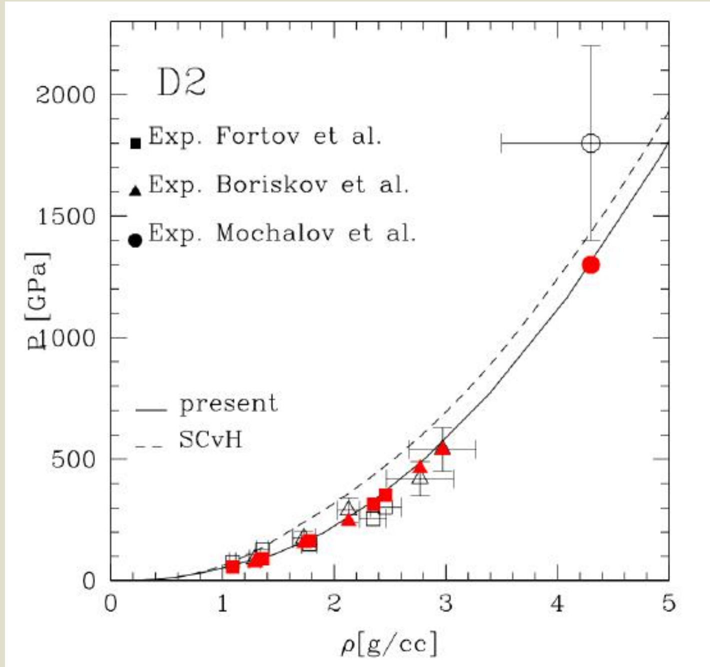
Prescribed a first order phase transition in Jupiter, now excluded

Improvements in 2020: interpolation with numerical results.

II. Metallic hydrogen







II. Metallic hydrogen

Conclusions on metallic H:

Most of the interior of giant planets is composed of metallic H with free, degenerated electrons (50% Saturn, 90% Jupiter)

Analytical solutions are untractable, but simplifications allow good prediction on radius, conductivity and evolution of giant planets

EOS need to couple theory, simulations and experiments

III. H/He immiscibility

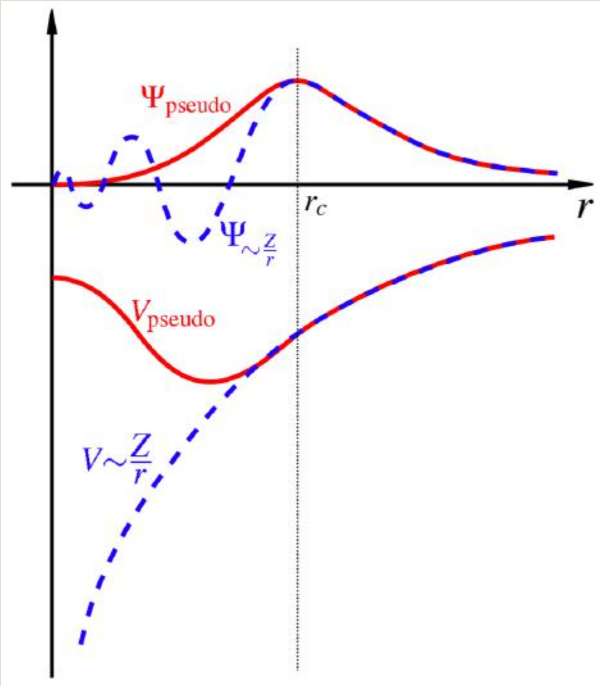
Theory: Kestner et al. 1965, Salpeter 1973, Stevenson 1975, Stevenson & Salpeter 1977a, b

Simulations: Lorenzen et al. 2011, Wilson & Militzer 2013, Morales et al. 2013, Schottler & Redmer 2018

No experiments yet …

III. H/He immiscibility

Original idea: from Kestner et al. 1965: strong short range repulsion between liquid helium and free electrons.



Pauli exclusion: interaction of bound and free electrons analytically difficult

Kestner et al. idea: prove the validity of the pseudo potential formalism on the He – electron interaction.

III. H/He immiscibility

The idea diffuses in the astrophysics literature, Smoluchowski, Salpeter … until Stevenson 1975

Like SCvH: calculation of the free energy of the H-He system.

$$F = F_{eg} + F_{hs} + E_M + E_{BS} + \Delta F_{int} + F_Q$$

Electron gaz, hard sphere (ions), Madelung, band structure, interaction correction, quantum temperature correction

III. H/He immiscibility

G and its derivatives: phase separation

$$\frac{\partial^2 G}{\partial x^2} < 0$$

Construction of the miscibility diagram

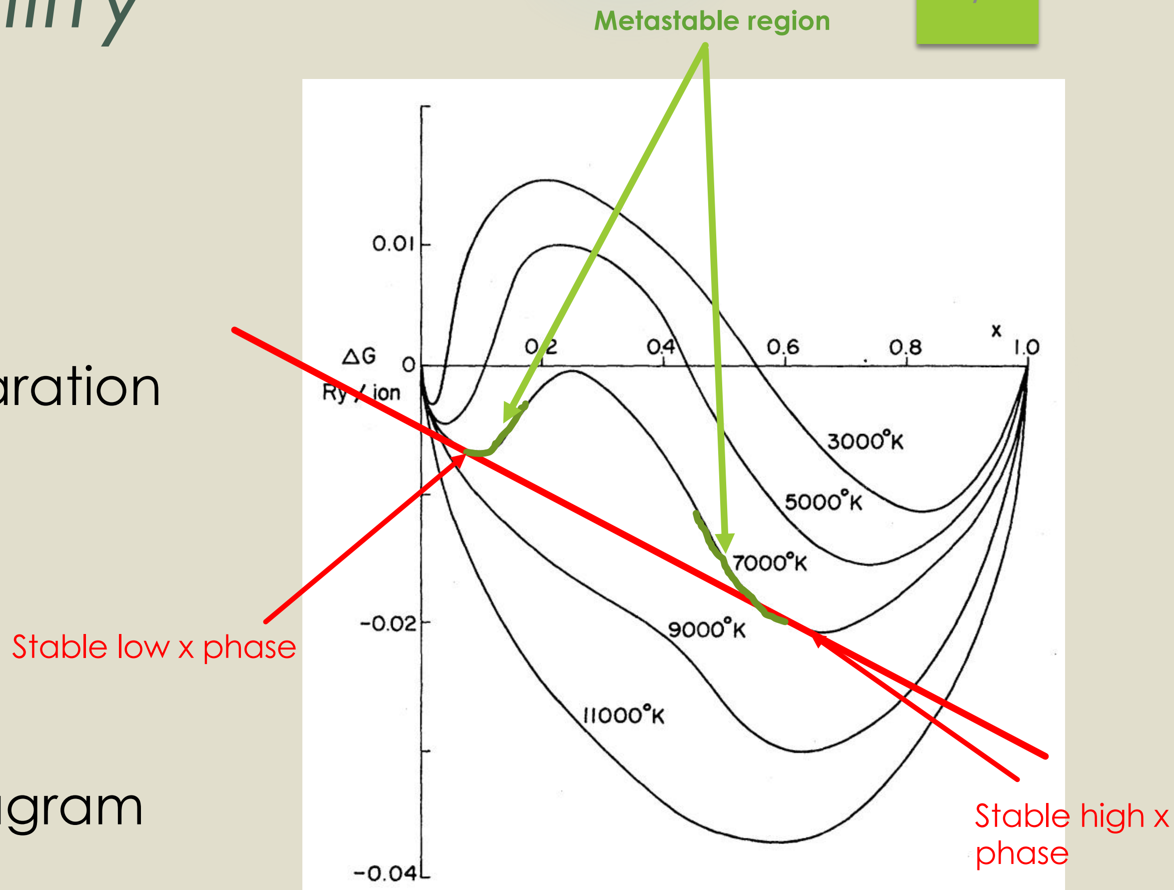
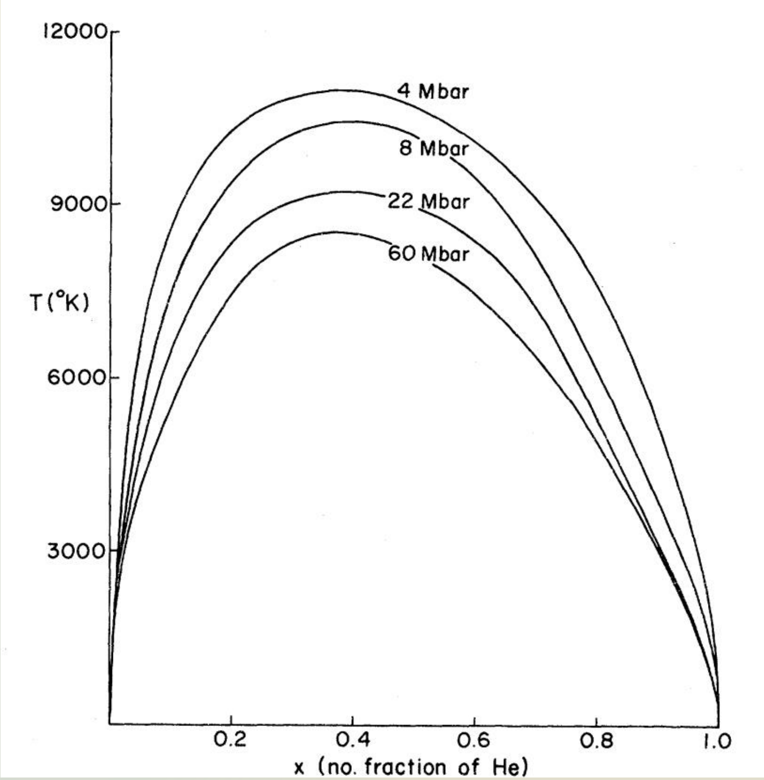


FIG. 2. Gibbs energy of mixing ΔG, as a function of T and x at $P = 8$ Mbar.

III. H/He immiscibility



Miscibility diagram

Immiscibility increases with decreasing temperature and pressure

Other noble gases ?

III. H/He immiscibility

Repulsive potential decreases with atomic number.

Wilson & Militzer 2010: simulations.
Ne immiscible in Jupiter, but not Ar and heavier noble gases

Table 1. Measured Mixing Ratios or Isotope Ratios

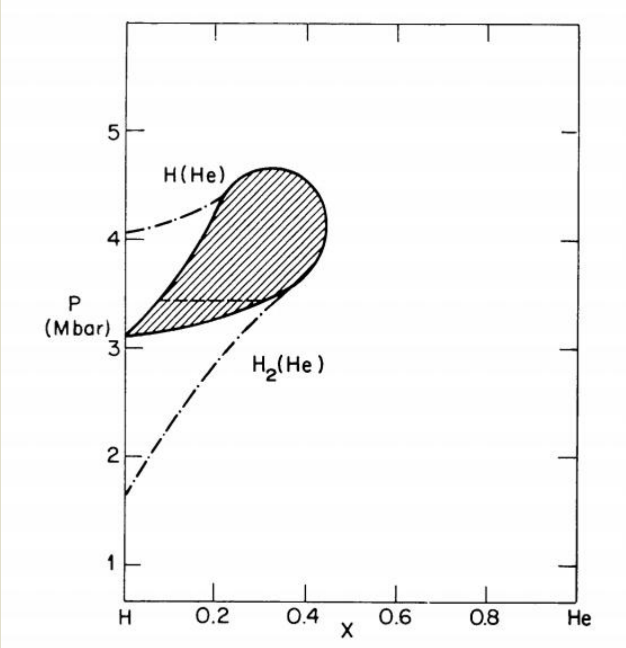
Species or Ratio	Mixing Ratio f or Isotope Ratio	Mole Fraction q	Ratio to Solar
^{4}He	0.157 ± 0.030	0.136 ± 0.026	0.8
$^{3}He/^{4}He$	$(1.66 \pm 0.05) \times 10^{-4}$		
D/H	$(2.6 \pm 0.7) \times 10^{-5}$		
^{20}Ne	$\leq 3 \times 10^{-5}$	$\leq 2.6 \times 10^{-5}$	≤ 0.13
^{36}Ar	$\leq 10.5 \times 10^{-6}$	$\leq 9.06 \times 10^{-6}$	≤ 1.7
^{84}Kr	$\leq 3.7 \times 10^{-9}$	$\leq 3.2 \times 10^{-9}$	≤ 5
^{132}Xe	$\leq 4.5 \times 10^{-9}$	$\leq 3.8 \times 10^{-10}$	≤ 5
H_2O			
3.6 bars	$\leq 8 \times 10^{-7}$	$\leq 6.9 \times 10^{-7}$	$\leq 4.1 \times 10^{-4}$
12 bars	$\leq (5.6 \pm 2.5) \times 10^{-5}$	$\leq (4.8 \pm 2.1) \times 10^{-5}$	≤ 0.033
19 bars	$\leq (6 \pm 3) \times 10^{-4}$	$\leq (5.2 \pm 2.6) \times 10^{-4}$	≤ 0.35
CH_4	$(2.10 \pm 0.4) \times 10^{-3}$	$(1.81 \pm 0.34) \times 10^{-3}$	2.9
$^{13}C/^{12}C$	0.0108 ± 0.0005		
NH_3 (>15 bars)	$\leq 2.3 \times 10^{-3}$	$\leq 2 \times 10^{-3}$	≤ 10
H_2S			
3.6 bars	$< 10^{-6}$	$< 8.6 \times 10^{-7}$	< 0.03
8.7 bars	7×10^{-6}	6.1×10^{-6}	0.23
>16 bars	$(7.7 \pm 0.5) \times 10^{-5}$	6.7×10^{-5}	2.5
PH_3 (>16 bar)	$\leq 6 \times 10^{-6}$	$\leq 5.2 \times 10^{-6}$	≤ 8
Cl	detected		

III. H/He immiscibility

What are the consequences of the H/He immiscibility ?

Stevenson & Salpeter 1977a, b

Gravity only a second order effect: the interior of giant planets is to be understood thermodynamically



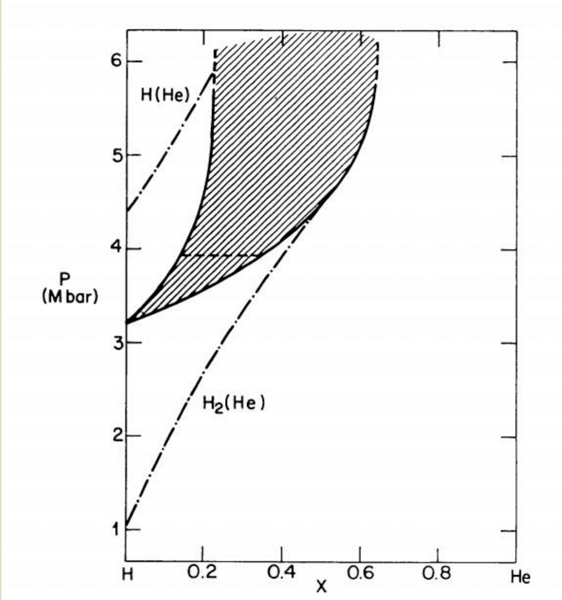
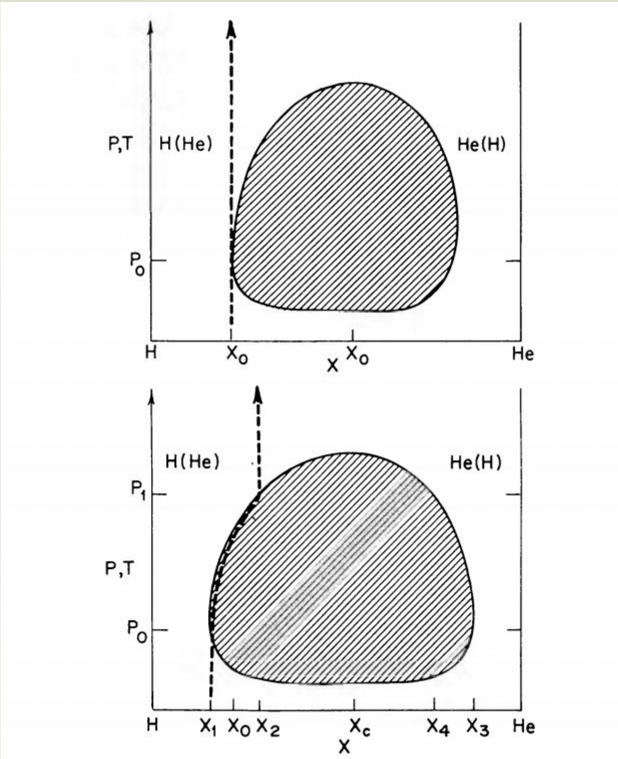
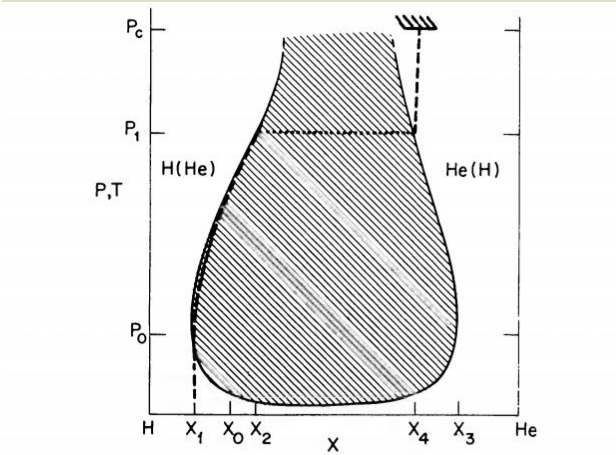


FIG. 5b

III. H/He immiscibility

Evolution





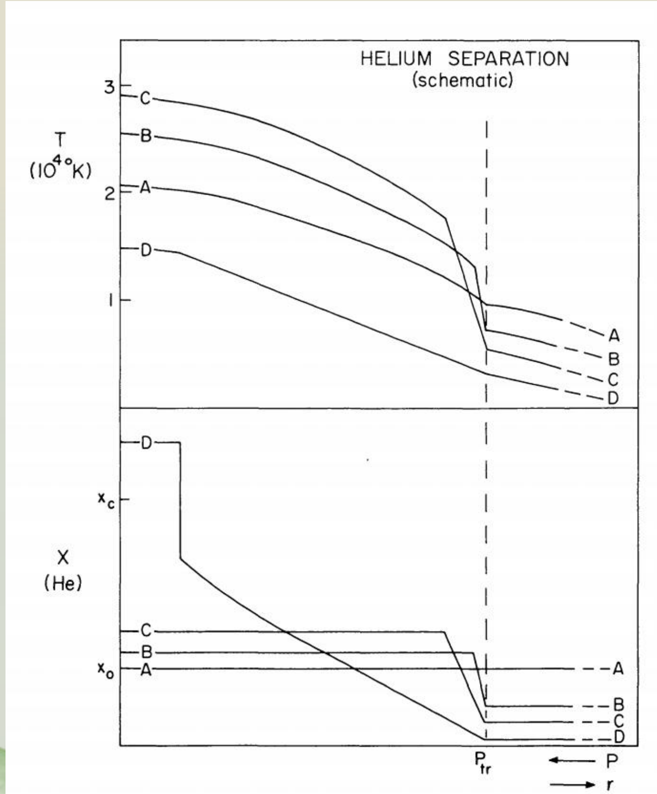
Depletion of the outer parts

Enrichment of the innermost part

Eventually: creation of a helium dominated core

III. H/He immiscibility

Evolution



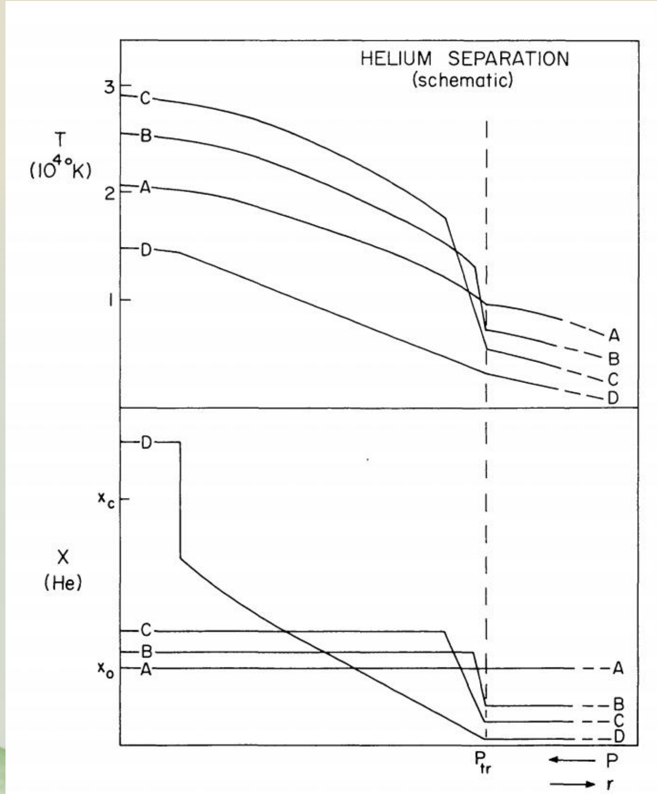
The He bubble release gravitational energy as they sink

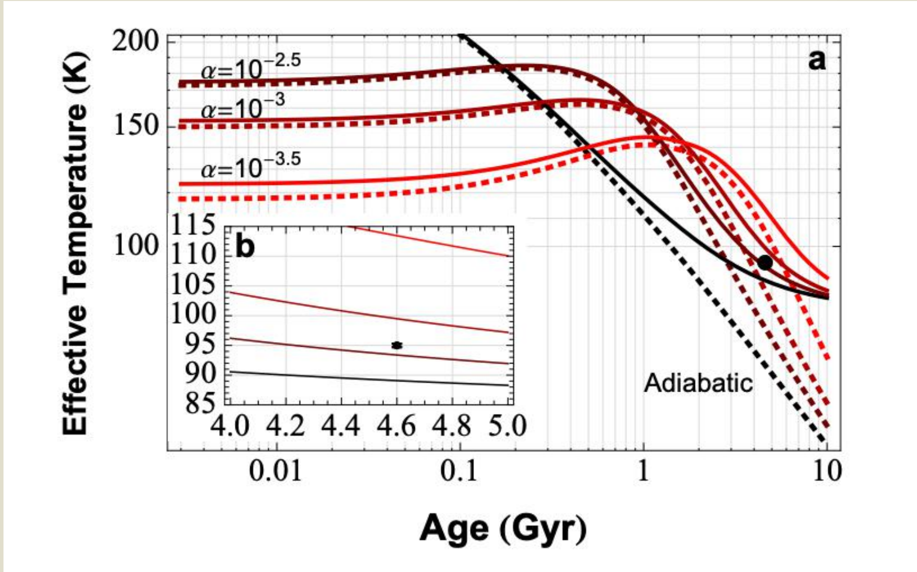
Interior heating up, cooling time increases

Explains Saturn's luminosity if He has decreased by ~ 20 %

III. H/He immiscibility

Evolution

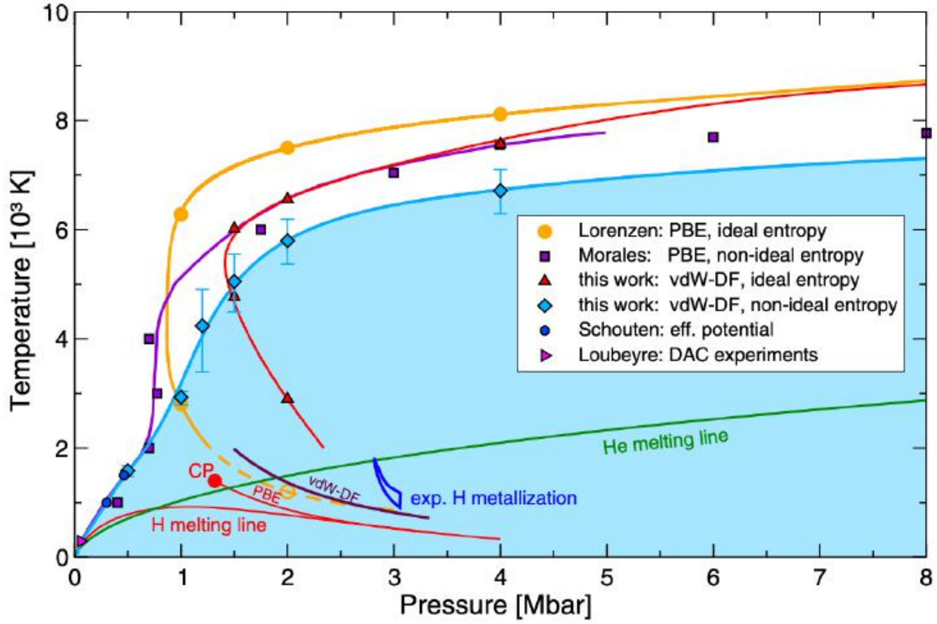


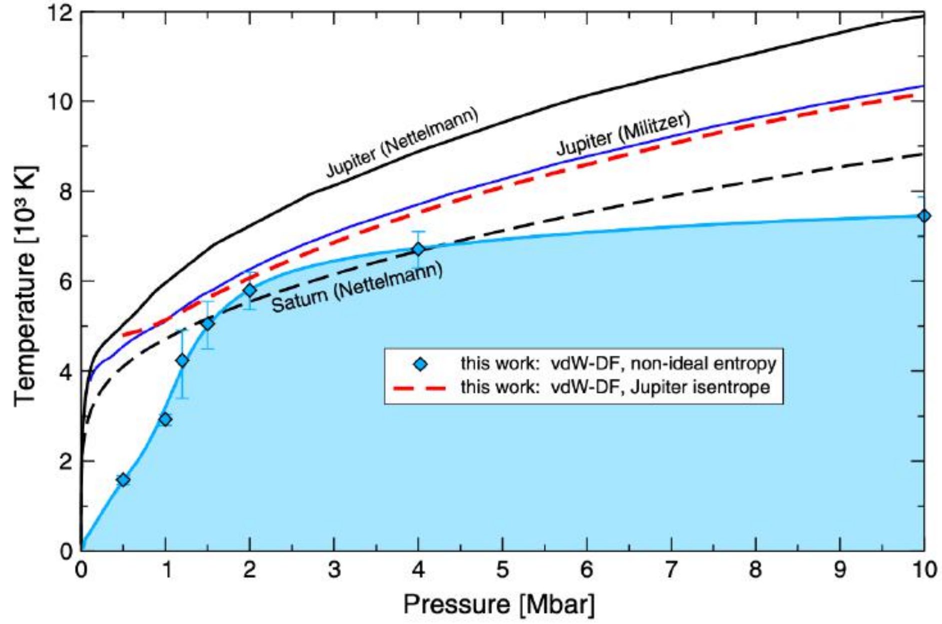


Layered convection. Leconte & Chabrier 2013

III. H/He immiscibility

What do we know today about H/He immiscibility ?





Schottler & Redmer 2018

III. H/He immiscibility

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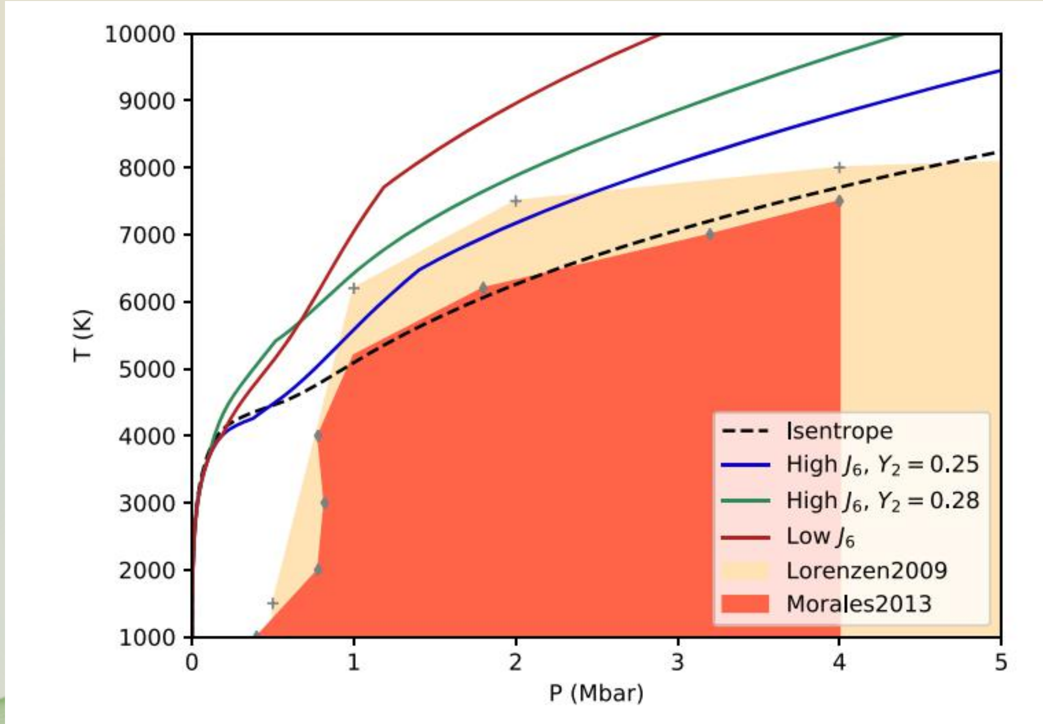

Debras & Chabrier 2019

How to explain He and Ne depletion ?

Immiscibility triggered semi convection ?

III. H/He immiscibility

Conclusions on immiscibility:

Strong repulsion of free electrons by He

Leads to immiscibility between metallic H and He at low temperature

Crucial consequences on the evolution of Jupiter and Saturn

Still out of reach experimentally !

Conclusions

Metallic hydrogen is the key constituent of giant planets

No analytically tractable EOS

Strong demixing with He, probably Ne. Other constituants ? Maybe

Improvements in experimental and numerical physics will change our understanding of giant planets

Thank you !