Theoretical Schema to Detect Quantum Nature of GWs using Quantum Systems

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INTRODUCTION

Recent observations of the gravitational waves (GWs) opened a new window to measure the exotic gravitational processes, using GWs as a new messenger capable to manifest the entire Universe, even the remnants of the big bang [1].

Existence of the primordial gravitational waves (PGWs) background is one of the crucial predictions of the inflationary scenario of the early Universe and its detection will provide us a hitherto inaccessible view of the Universe. PGWs span a full range of frequencies and are fortunately reachable by either current GW detectors such as LIGO or future experiments such as the planned laser interferometer space-based antenna (LISA).

On top of that, it is believed that PGWs generated by the strong gravitational pumping engine of the very early Universe had evolved into the so-called squeezed states, and after that, they decoupled from the rest of matter and radiation, and freely propagated throughout the Universe [2]. Detecting the non-classicality of the PGWs would not only provide us valuable information about the quantum mechanical origin of the Universe but also affirm the quantum nature of gravity.

There have been proposed various theoretical schemes to search for the non-classical nature of PGWs and in particular their squeezed essence [3]. On the experimental side, however, the search for demonstrations of the quantum nature of PGWs is still challenging.

We develop a new theoretical framework based on optical medium analogy (OMA), within which it's possible to track the effect of classical (quantum) features of PGWs on the classical (quantum) observables of the EM field.

Classical Treatment

GWs are solutions to the linearized Einstein equations of a perturbed spacetime metric. We perform our description in traceless transvers (TT) gauge and keep only linear terms in the perturbation to the flat space-time metric (linearized gravity). Thus the metric perturbation can be described by

$$h_{ij}(t)=\sum_{\gamma=+,\times}\int\frac{d^3K}{(2\pi)^{3/2}}\hat{e}_{ij}[\hat{K}]\left(h_\gamma(\vec{K})e^{-i(\Omega t-\vec{K}\cdot\vec{r})}+c.c.\right) \quad (1)$$

In which we have considered the ***"small detector"*** condition, i.e., $\lambda_{gw}\ll L$. In the OMA formalism [4], the GW background manifests itself as a non-dispersive non-absorptive magneto-dielectric media, possessing anisotropic magneto-dielectric permittivity tensors

$$\varepsilon_{ij}(t)=\mu_{ij}(t)=\delta_{ij}-h_{ij}(t) \quad (2)$$

Having the Maxwell's equations in the magneto- dielectric media, the vector potential expansion in terms of the mode functions turns out to be

$$\vec{A}(\vec{r},t)=\sum_{\nu,k}\sqrt{\frac{\hbar}{2m(t)\omega_k(t)}}\left(\alpha_k\hat{u}_k e^{-i(\int_0^t\omega_k(t')dt'-\vec{k}\cdot\vec{r})}+c.c.\right) \quad (3)$$

Where $m(t)=\varepsilon_{ij}(t)u_iu_j$, $\omega_k(t)=\omega_k(t_0)/n(t)$ and $n(t)$ is the refractive index of the medium. For an EM signal propagating in the *x-z* plane, making angle θ with z-axis:

$$n(t)=1-\frac{1}{2}\left(h_{11}(t)\cos^2\theta+h_{22}(t)+h_{33}(t)\sin^2\theta-h_{13}(t)\sin 2\theta\right) \quad (4)$$

At the *classical level*, besides recovering the previously known response function of a LISA-like interferometer, we investigate the (i) walk-off angle of light which can be interpreted as the ***light-bending*** angle as well as (ii) the ***Stokes parameters*** of the EM signal.

The ***light-bending*** angle turns out to be proportional to the strain field of the GWs background and is given by

$$\rho(t)=\frac{1}{2}\left(h_{11}(t)-h_{33}(t)\right)\sin 2\theta+h_{13}(t)\cos 2\theta \quad (5)$$

To determine the polarization state of light we investigate the Stokes parameters, since they are quadratic in the field strength and can be determined through intensity measurements only. It turns out that a linearly polarized light stays linear but rotates about y-axis with angle given by *Eq.(5)* (see *Fig. 1*).

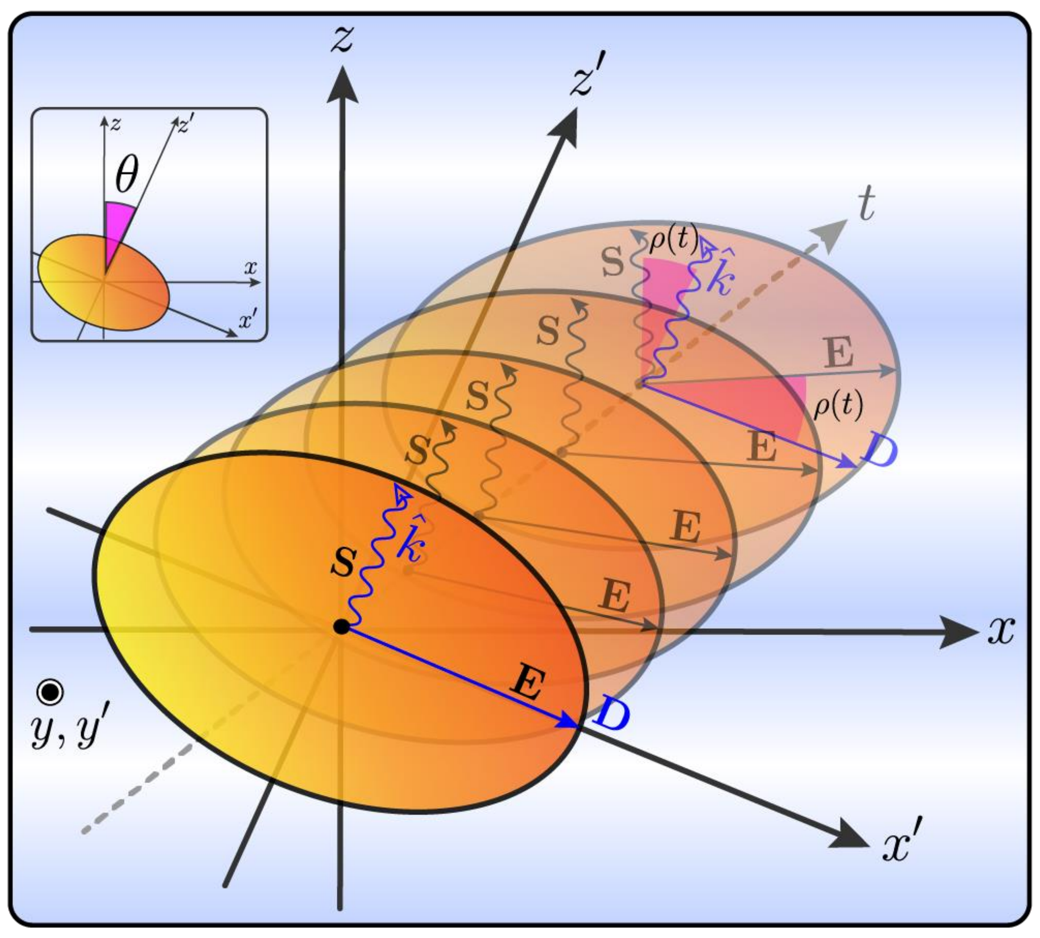

Figure 1. Light-bending and polarization ellipse of EM field, in the presence of the GWs background. As light propagates along *z'* direction, the Poyinting vector **S** and electric vector **E**, start to rotate about *y-axis*, which results in the light bending, determined by Eq.(5).

Quantum mechanical Treatment

In order to reveal the quantum nature of GWs, it's necessary to promote description to a fully quantum mechanics one. Within the OMA approach, this is accomplished following the standard canonical quantization framework. We show that at the quantum level, the effect of GWs on the EM field is to change it's eigen-frequencies as

$$\hat{H}=\hbar\omega_k\hat{a}_k^\dagger\hat{a}_k\rightarrow\hat{H}(t)=\frac{\hbar\omega_k}{n(t)}\hat{a}_k^\dagger\hat{a}_k \quad (6)$$

Where n(t) is the refractive index. Hence, the *quantized Hamiltonian* describing the interaction between quantized GWs and quantized EM field resembles the well-known *optomechanical coupling,* and is given by

$$\hat{H}_{\text{int}}(t)=-\frac{1}{2}\hbar\omega_k\left(\hat{h}_{11}\cos^2\theta+\hat{h}_{22}+\hat{h}_{33}\sin^2\theta-\hat{h}_{13}\sin 2\theta\right)\hat{a}_k^\dagger a_k \quad (7)$$

Where $\hat{h}_{ij}(t)$ is the quantized weak GWs given by *Eq.(1)* with substituting $h_\gamma(\vec{K})\rightarrow\sqrt{16\pi c}\,l_{Pl}\,\hat{b}_K$ and $\hat{b}_K,\hat{b}_K^\dagger$ are the bosonic annihilation and creation operators for the *K-th* mode of the GWs background.

The Hamiltonian *Eq.(7)* is reminiscent of the ***opto-mechanical coupling*** of a massive mirror to light, induced by the radiation pressure [5]. This coupling can be viewed virtually as an intensity dependent "displacement" of GWs, very similar to the mechanical motion of a massive mirror induced by the radiation pressure in a cavity opto-mechanical system. The optomechanical analogy becomes more evident with the association ***GW strain field ~ mechanical oscillator displacement field***.

In a similar manner, we call this interaction ***"opto-gravitational" coupling***. It turns out that the mutual interaction between EM and GWs, not only induce EM field dynamics, but also the radiation pressure of the light induces the strain field dynamics.

DISCUSSION

Quantum dynamics of both EM field and GWs are determined by the Hamiltonian *Eq.(7)*. By deriving the unitary evolution operator, one can see that the EM field annihilation operator obeys

$$\hat{a}(t)=e^{iE(t)}e^{-\int d^3K\,\kappa(\Omega)\left(\hat{b}_K^\dagger\eta_K(t)-\hat{b}_K\eta_K^*(t)\right)}e^{2iE(t)\hat{a}^\dagger a}\hat{a} \quad (8)$$

where the effect of GWs is encoded in the time dependent functions $E(t),\kappa(\Omega),\eta_K(t)$. Using this expression, one can calculate the time behaviour of different observables of the EM field.

The effective opto-gravitational coupling imply that measurements on the EM field not only can reveal properties of gravitons, including their non-classicality, but also offers a rout toward fundamental tests of quantum mechanics in a yet inaccessible parameter regime of frequency and amplitude of oscillations, associated to GWs.

It is desirable to come up with witnesses of gravitational non-classicality, by studying pure quantum mechanical effects.

For example, In the context of opto-mechanics, it has been shown that "revivals of squeezing" is a pure quantum mechanical effect [6], in the sense that it exist only if both the EM field and mechanical oscillator are treated as quantum entities.

Thus, in the context of opto-gravitational coupling of light and GWs, although the squeezing of light can not be an evidence of non-classicality of GWs, one can search for the quantum mechanical origin of GWs by investigating the variance of quadrature components of the EM field, to see the revivals of squeezing.

One can also try to see the effect of different states of GWs on the quantum observables of EM field. In particular, it is interesting to study the imprint of squeezed PGWs, on the quantum correlation functions of EM field. For example, the Hanbury Brown–Twiss interferometry of light may reveal quantum correlations present in the GWs Background.

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