Helical Magnetorotational Instability in Strongly Stratified Radiative Zones

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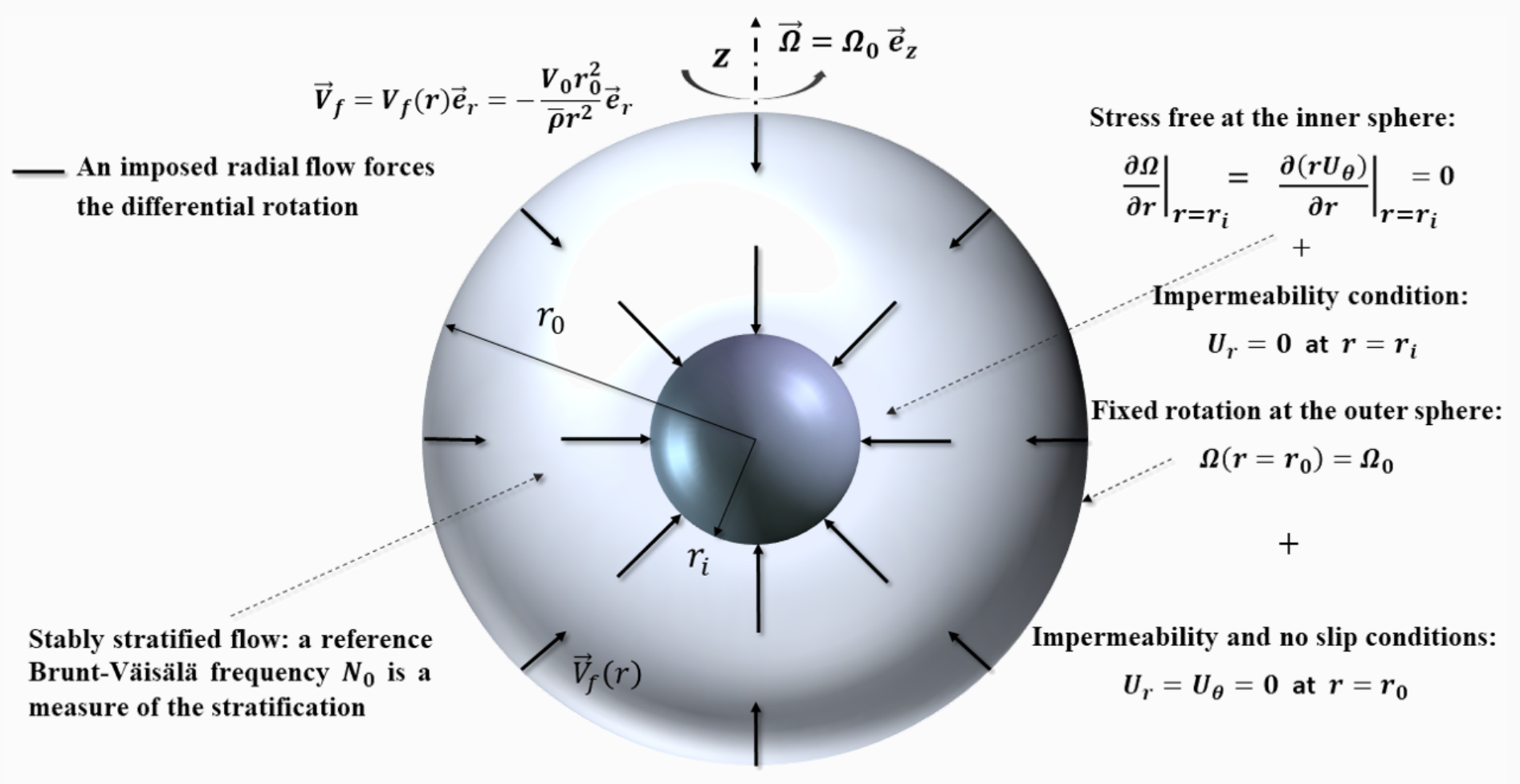
1. Astrophysical Context

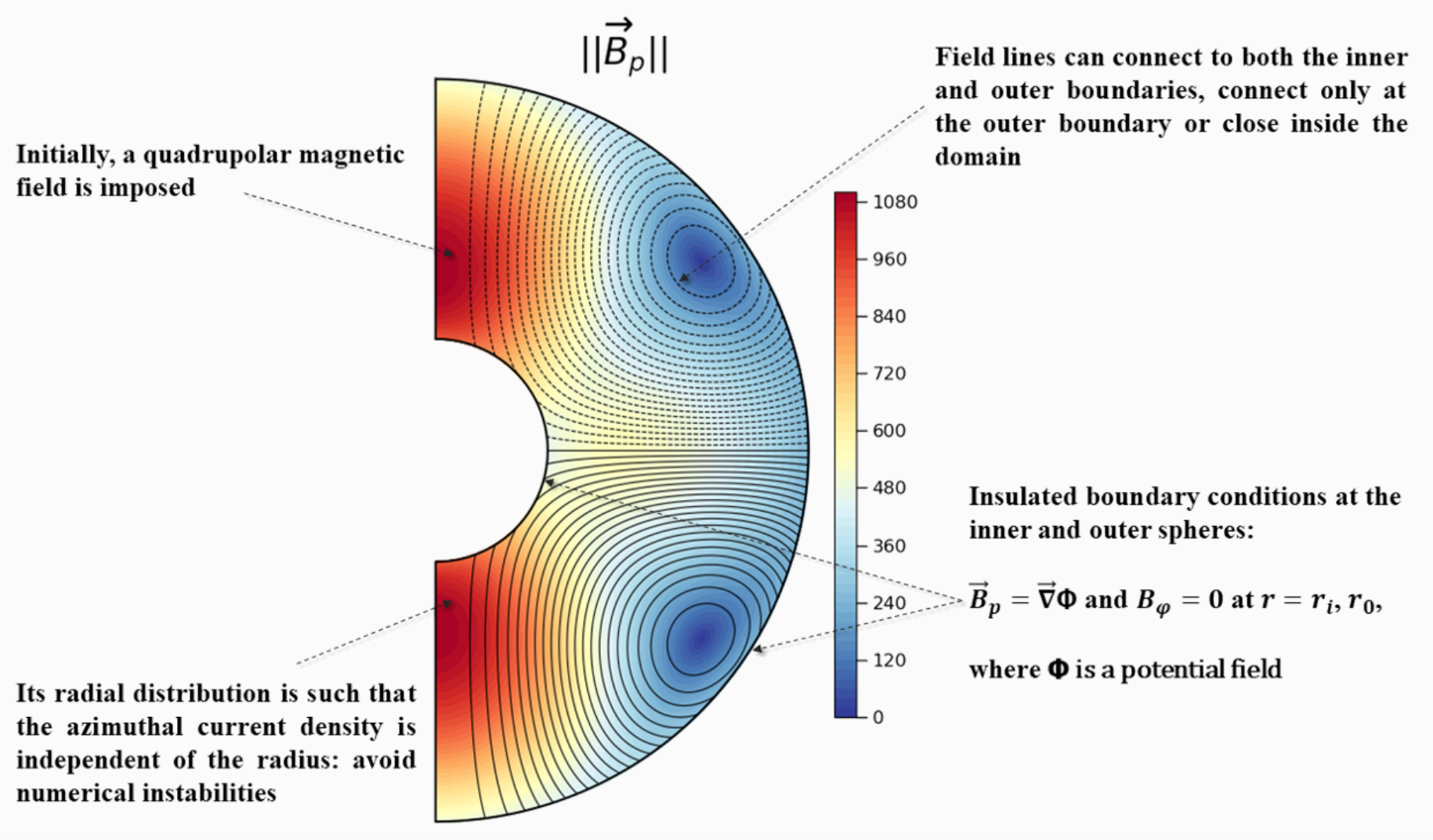
When a low-mass star has exhausted its hydrogen content, it leaves the main sequence (MS) and its core experiences a gravitational contraction. During this collapse, the star heats up and hydrogen fusion can start in a shell surrounding the inert helium core. While above this H-burning shell the star expands, below, its core contracts and becomes degenerate. Asteroseismic measurements of the rotation rates of these post-MS stars show that their cores spin-up as a consequence of their contraction. However, they are found to rotate orders of magnitude slower than what is expected from pure hydrodynamical stellar evolution models. An efficient transport of angular momentum (AM) thus seems to be at play during the contraction phase. Interestingly, some of these stars are thought to host a large-scale magnetic field in their radiative zone. Such fields could be produced by a dynamo existing in their convective core during the MS. The contraction-induced differential rotation could then potentially interact with the magnetic field, thus producing various magneto-hydrodynamical instabilities. In the present work we show that a helical magnetorotational instability (HMRI) can be triggered in strongly stratified radiative zones such as the degenerate cores of these post-MS stars, in the presence of a quadrupolar field. Such an instability is of particular interest since it is able to destroy the large-scale structure of the magnetic field, then enabling the differential rotation to set in. By doing so, it thus constitutes a promising way to explain the evolution of the rotation as observed by asteroseismology.

2. Setup

Characteristic timescales:

- One is related to the contraction $\tau_c = \dfrac{r_0}{V_0}$.
- Two are associated with an AM transport carried out either by viscous processes on $\tau_\nu = \dfrac{r_0^2}{\nu}$, or a large-scale meridional circulation on $\tau_{\mathrm{ED}} = \dfrac{r_0^2}{\kappa}\left(\dfrac{N_0}{\Omega_0}\right)^2$.
- Two timescales are due to the presence of a magnetic field $\tau_\eta = \dfrac{r_0^2}{\eta}$ and $\tau_{\mathrm{A_p}} = \dfrac{r_0\sqrt{\mu_0\rho_0}}{B_0}$.





Dimensionless numbers:

$$E = \frac{\nu}{r_0^2\Omega_0}\ ;\quad P_r = \frac{\nu}{\kappa}\ ;\quad \left(\frac{N_0}{\Omega_0}\right)^2\ ;$$

$$Re_c = \frac{V_0 r_0}{\nu}\ ;\quad L_u = \frac{B_0 r_0}{\sqrt{\mu_0\rho_0}\eta}\ ;\quad P_m = \frac{\nu}{\eta}$$

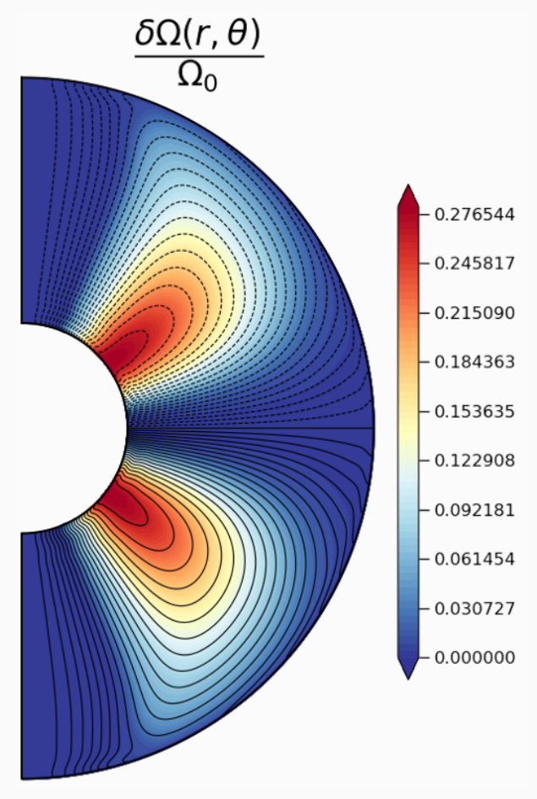
Regime of interest: the viscous regime

$$\tau_{\mathrm{A_p}} \ll \tau_c \leq \tau_\nu \ll \tau_\eta \ll \tau_{\mathrm{ED}}$$

or equivalently,

$$\left(\frac{L_u}{P_m}\right)^{-1} \ll Re_c^{-1} \leq 1 \ll P_m \ll P_r\left(\frac{N_0}{\Omega_0}\right)^2.$$

3. Typical state after $\sim 1\,\tau_c$



Two magnetically decoupled regions.

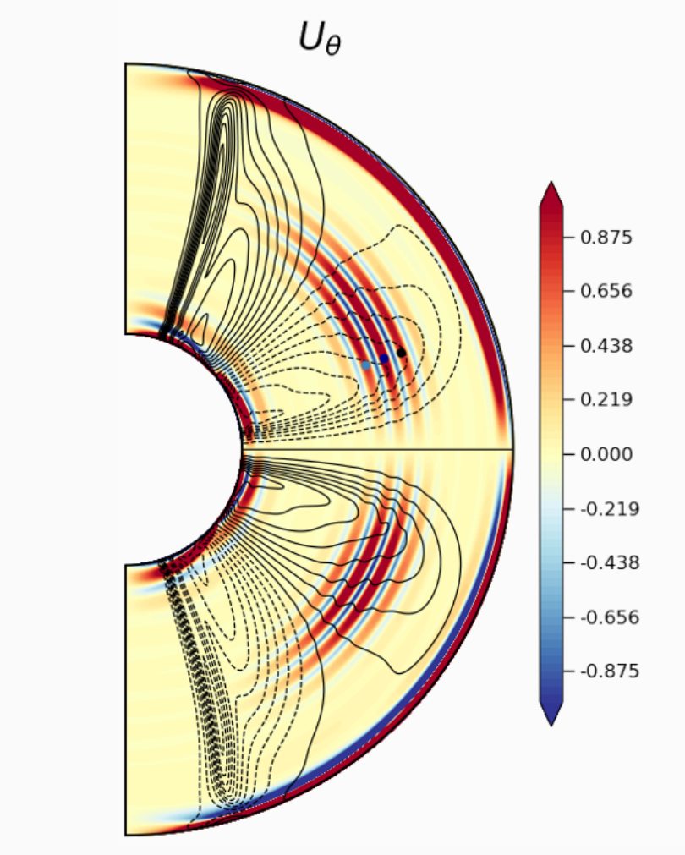
- A quasi-solid rotation region where:

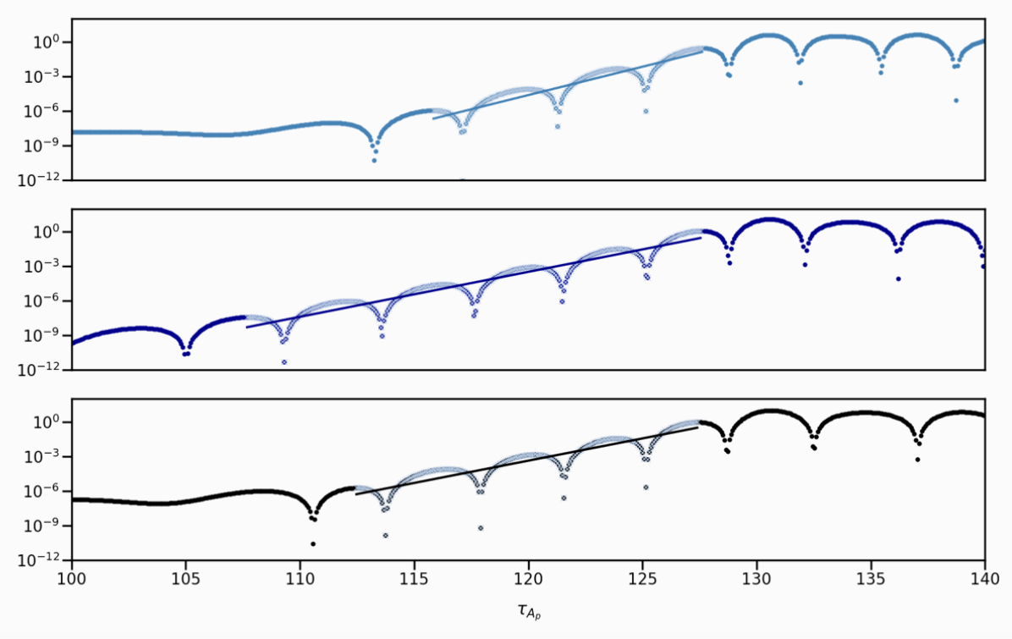
$$\underbrace{-2\sin\theta\,\Omega_0\frac{V_0 r_0^2}{r^2}}_{\text{Contraction term}} = \underbrace{\frac{1}{\mu_0\rho_0}\left(\vec{B}_p\cdot\vec{\nabla}\right)B_\phi}_{\text{Lorentz force}}.$$

- A dead zone where the differential rotation is given by:

$$\underbrace{-2\sin\theta\,\Omega_0\frac{V_0 r_0^2}{r^2}}_{\text{Contraction term}} = \underbrace{\nu\vec{\nabla}^2 U_\phi}_{\text{Viscous term}}.$$

4. Nature of the instability





MRI-type instability [1]:

The top figure shows that the perturbations grow at the location where $\partial\ln\Omega/\partial\theta < 0$ in the northern hemisphere ($\partial\ln\Omega/\partial\theta > 0$ in the southern hemisphere).

They are radially confined and develop preferentially in the latitudinal direction.

Growth rates are $\propto |\partial\ln\Omega/\partial\theta|$.

The poloidal field has a stabilising effect.

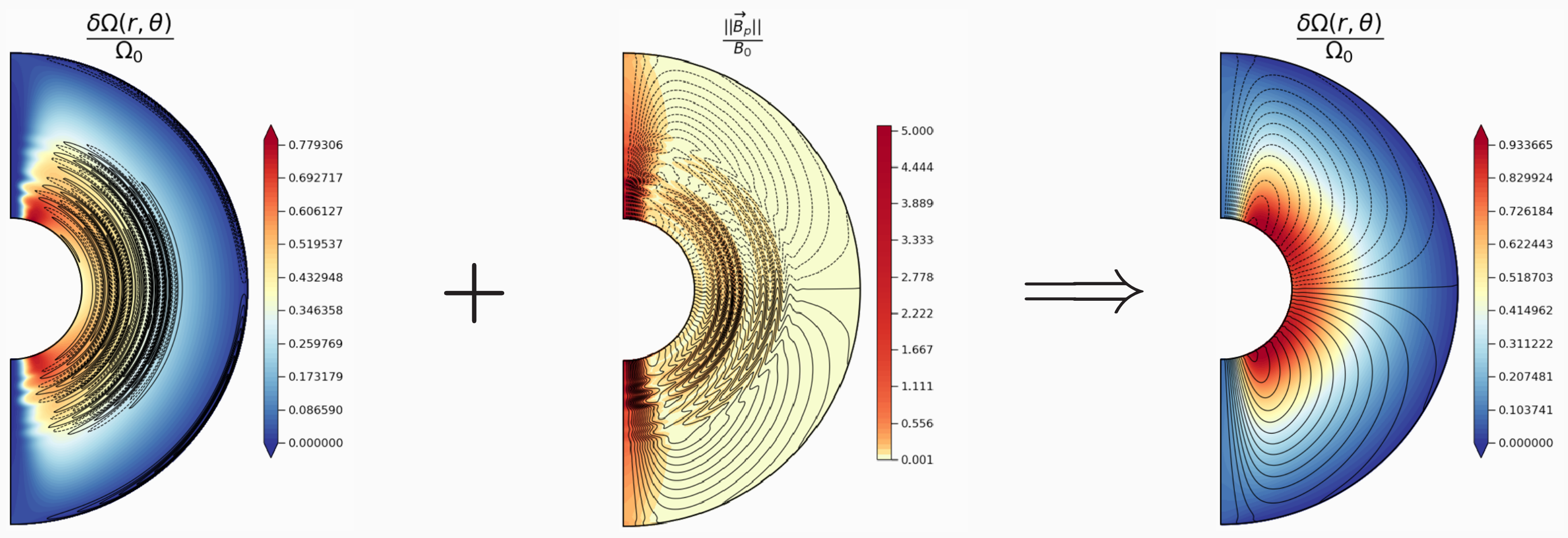
HMRI instability:

The toroidal field is stabilising [2].

Non-stationary character of the unstable modes (bottom figure) [3].

Phase drift modifying the phase dependency between the perturbed fields [4].

5. Non-linear evolution and flow reconfiguration



The poloidal field (second panel) behaves like a passive scalar advected and mixed by a multi-cellular meridional circulation (first panel). This creates small scales on which it is efficiently dissipated.

The Lorentz force is no longer active between the two dead zones: the contraction is balanced by the viscous effects in a larger domain and the differential rotation increases (third panel).

6. Conclusion

During the post-MS evolution, the envelope of a low-mass star expands while its core contracts and accelerates. However, asteroseismic measurements tend to show that during the subgiant phase a mechanism prevents the core acceleration, sometimes producing rotation profiles compatible with a solid-body rotation [5]. After this phase, this mechanism seems to be less effective since the core tends to spin-up again. The instability described in this work therefore enables us to consider a physical scenario in which an almost solid-body rotation is maintained during a contraction timescale by a large-scale magnetic field. Then, the differential rotation built up inside the dead zones becomes sufficient to trigger an instability which destroys this large-scale structure, letting the contraction accelerate the core again and thus produce a significant differential rotation. At later stages (e.g., giant phase), other processes of AM transport have to be invoked to explain the observed rotation rates.

References

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