Wave Chaos in Rapidly Rotating Stars

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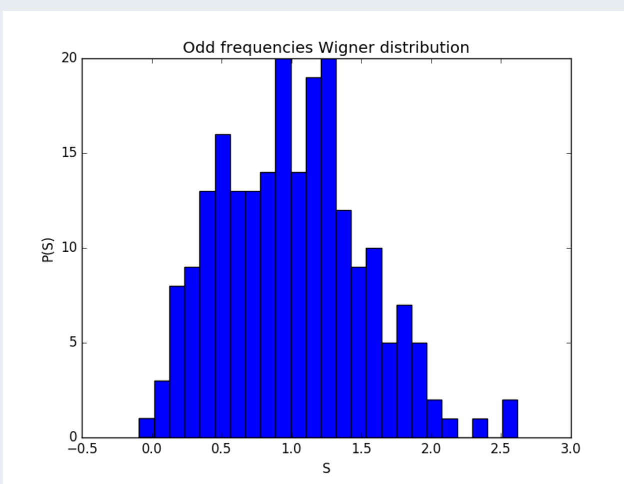
Universality in chaotic systems

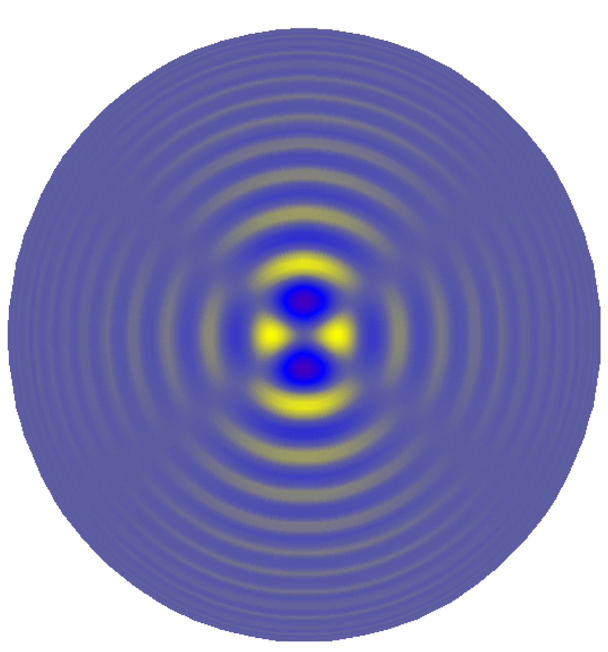
The shape of the spacing distribution is roughly the same for all chaotic spectra.



Spacing distribution

Asteroseismology

By studying the oscillations of a star we can gain information about its interior : that is the goal of asteroseismology.
In this thesis I will focus on oscillations induced by pressure/acoustic waves (p-modes).

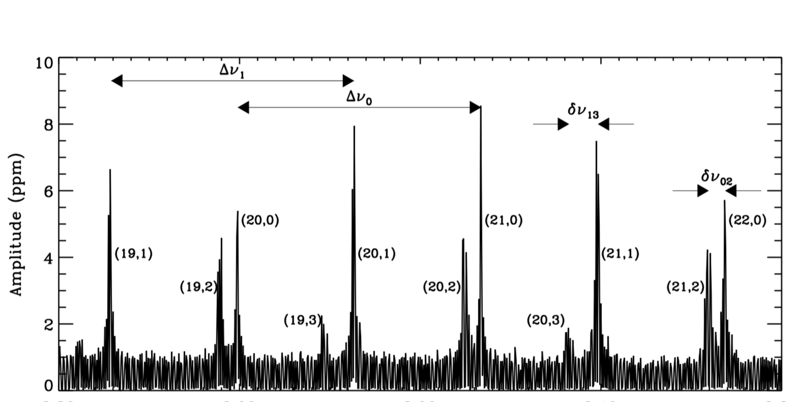


oscillation mode of a non rotating star

In the case of slowly rotating stars, the spectrum of acoustic oscillations is well discribed in the high-frequency regime by Tassoul's asymptotic formula :

$$\omega_{n,l} = \Delta\omega\left(n + \frac{\ell}{2} + \frac{1}{4} + \alpha\right)$$
$$\Delta\omega = 2\pi\left(2\int_0^{R_e}\frac{dr}{c_s}\right)^{-1}$$

This formula relates the great separation $\Delta\omega$, a quantity we can access through observations, to the speed of sound c_s in the star.



Solar oscillations

Effect of Rotation

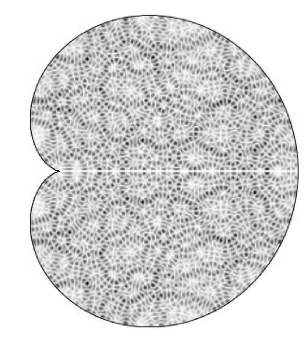
At increased rotation we see new types of p-modes [1] :

- whisperring gallery modes
- 2-period island modes
- 6-period island modes
- chaotic modes

Work has been done to understand the asymptotic structure of the ilsand modes spectrum [2]. The goal of my thesis is to study the spectrum of chaotic modes.

Quantum Chaos

Quantum chaos is the study of quantum systems whose classical limit is chaotic.

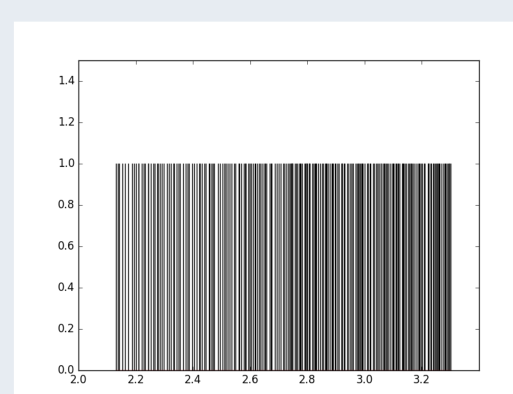


stationnary wavefunction inside a cardioid cavity

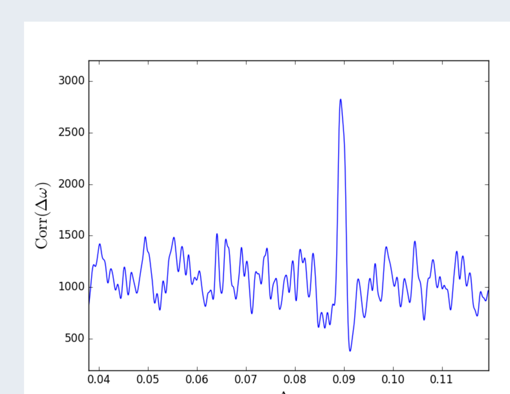
chaotic wavefunctions are ergodic : the probability density is rather uniformly distributed inside the cavity.

Regularities in the chaotic spectrum

The autocorrelation of the chaotic spectrum shows a big spike at the great separation. It is an unusual feature for a chaotic system.



Chaotic spectrum



Autocorrelation of the chaotic spectrum

Ray Dynamics

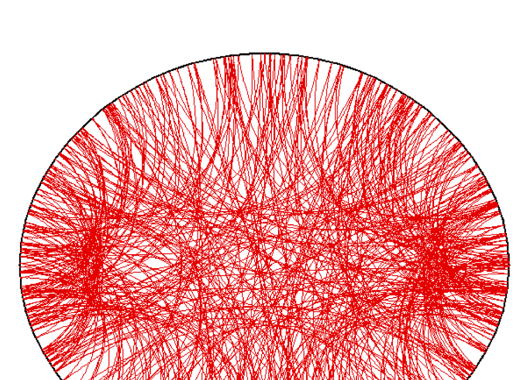
In the small wavelength approximation acoustic waves propagate according to the eikonal equation :

$$\omega^2 = \omega_c^2 + c_s^2 k^2$$

From this we derive the Hamiltonian :

$$H = \frac{\mathbf{k}^2}{2\omega^2} + W$$
$$W = -\frac{1}{2c_s^2}\left(1 - \frac{\omega_c^2}{\omega^2}\right)$$

The potential barrier at the surface, responsible for the back-reflection of the wave, stems from the sharp increase of the cutoff frequency ω_c.



Chaotic trajectory of an acoustic ray

An important result of quantum chaos, that is valid for bounded systems, is the Gutzwiller formula :

$$d(E) = \bar{d}(E) + \frac{1}{\hbar}\sum_j A_j\, e^{i\left(\frac{S_j(E)}{\hbar} + \nu_j\right)}$$

The sum is calculated over the periodic orbits : S_j is the action along the jth orbit and A_j is linked to the stability of the jth orbit.

Having an expression for the density of the spectrum might seem enough to calculate other statistical quantities, such as variance, but the series is divergent and therefore hard to manipulate.

Michael Berry showed [3] that the fourier transform $K(T)$ of the correlation function $<[d(e-\frac{1}{2}\xi)-1]\,[d(e+\frac{1}{2}\xi)-1]>$ can be approximated as :

$$K(T) = \frac{1}{\hbar\bar{d}}\sum_j A_j^2\,\rho$$

Where ρ is the distribution of actions along periodic orbits.

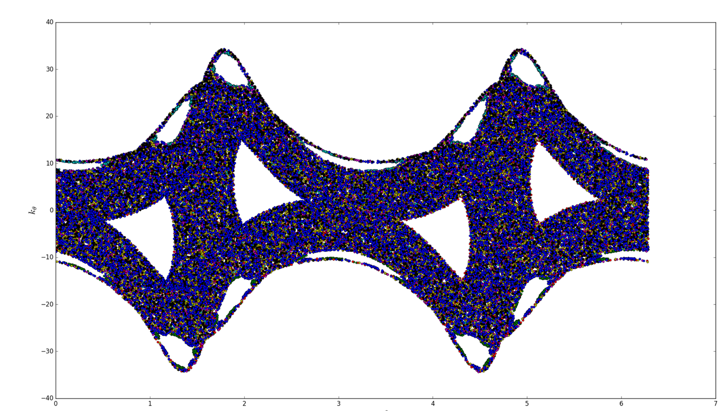
Action Along Ray Paths

The results of quantum chaos remain true outside of quantum physics, with :

$\hbar \to \omega^{-1}$
$S \to T$, where T is the travel time of a ray along a certain path.

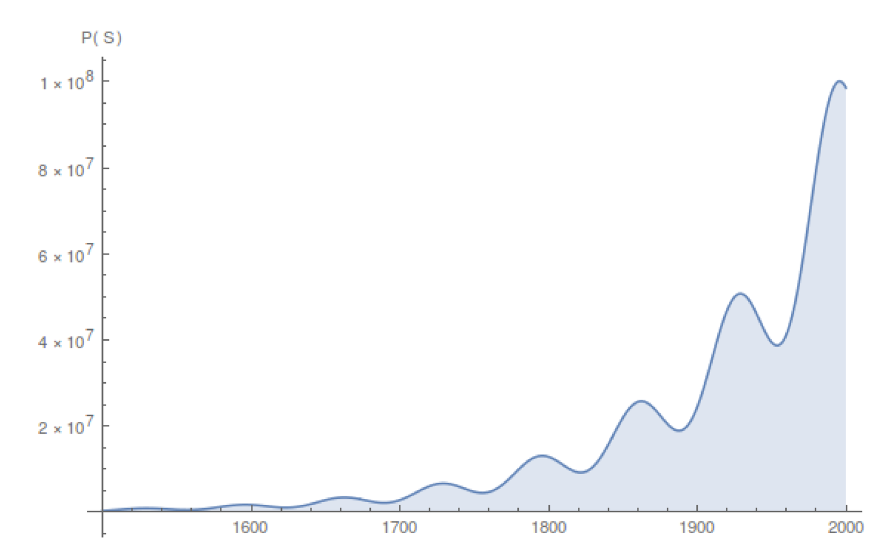
Now we want $\rho(T)$.

To find chaotic paths, we look at the Poincaré surface of section. For long times, a single chaotic trajectory will fill up a region of phase space.



Poincaré surface of section : chaotic zone

Keeping only periodic orbits, we end up with the following distribution $\rho(T)$:



Distribution of the actions along periodic orbits

Conjecture : the oscillation of $\rho(T)$ is at the origin of the chaotic spectrum correlations.

References

[1] F. Lignières, B. Georgeot. Asymptotic analysis of high-frequency acoustic modes in rapidly rotating stars A&A 2009

[2] M. Pasek, F. Lignières, B. Georgeot, D. R. Reese. Regular oscillation sub-spectrum of rapidly rotating stars A&A 2012

[3] M. Berry Some quantum to classical asymptotics