$\Omega\Gamma$-limit and anisotropic mass loss from rapidly rotating early-type stars

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1. Introduction

In stars more massive than $\sim 7M_\odot$ radiative acceleration plays a significant role in the (assumed) hydrostatic equilibrium, and one usually introduces [e.g. 6] the total gravity

$$\boldsymbol{g}_{\rm tot} = \boldsymbol{g}_{\rm eff} + \boldsymbol{g}_{\rm rad}$$

where the effective gravity $\boldsymbol{g}_{\rm eff}$ is supplemented by the radiative acceleration

$$\boldsymbol{g}_{\rm rad} = \frac{\kappa \boldsymbol{F}}{c}$$

where κ is the total opacity.

- The $\Omega\Gamma$-limit is reached when $\boldsymbol{g}_{\rm tot} = \boldsymbol{0}$ somewhere on the stellar surface
- It is associated with a corresponding break-up angular velocity Ω_c
- For $\Omega > \Omega_c$ the star is no longer gravitationally bound

We use the ω-model to derive Ω_c as well as a local mass-flux prescription for rapidly rotating early-type stars.

2. Maeder (1999) generalisation of the von Zeipel theorem

There have been some debate in the literature about the expression of the correct critical angular velocity when the effects of radiation cannot be neglected. [5] found two roots to the equation $\boldsymbol{g}_{\rm tot} = \boldsymbol{0}$

- $\Omega_c = \Omega_k \equiv \sqrt{GM/R_e^3}$ for Eddington ratios $\Gamma < 0.639$
- $\Omega_c < \Omega_k$ for increasing $\Gamma > 0.639$ with $\Omega_c \to 0$ when $\Gamma \to 1$

Both [5] and [6] use the vin Zeipel theorem [8] which states that:

$$\boldsymbol{F}(\theta) \propto -\boldsymbol{g}_{\rm eff}(\theta)$$

In the case of shellular rotation ($\Omega \simeq \Omega(r)$) they develop linearly all quantities around their average on an isobar

$$\boldsymbol{F} = -\frac{L(P)(1-\zeta(\theta))}{4\pi GM}\left(1 - \frac{\Omega^2}{2\pi G\rho_m}\right)^{-1}\boldsymbol{g}_{\rm eff}\ . \qquad (1)$$

- According to this model $F/g_{\rm eff}$ has a rather mild dependence with θ ($\zeta(\theta) \ll 1$)
- This is in contradiction with both observations and 2D-ESTER models
- Interferometric observations of several rapidly rotating stars [e.g. 7, 9, 2, 3] show that

$$\boldsymbol{F}(\theta) \propto -\boldsymbol{g}_{\rm eff}(\theta)^{4\beta}\ ,$$

with $\beta < 1/4$ for all the observed stars.

- ESTER models show that $\beta \simeq 1/4$ but decreases as the rotation rate increases

This behaviour has implications on the $\Omega\Gamma$-limit that we shall now discuss using the simplified ω-model of [4].

3. The ω-model

The purpose of the ω-model is to describe the latitudinal variations of the radiative flux of rotating stars in a simpler way than with a full two-dimensional model. It yields

$$\boldsymbol{F}(r,\theta) = -\frac{L}{4\pi GM}\frac{\tan^2\psi(r,\theta)}{\tan^2\theta}\boldsymbol{g}_{\rm eff}$$

where ψ is obtained solving

$$\cos\psi + \ln\tan(\psi/2) = \frac{1}{3}\omega^2 r^3\cos^3\theta + \cos\theta + \ln\tan(\theta/2)\ .$$

The equatorial radiative flux reads

$$\boldsymbol{F}(R_{\rm eq},\pi/2) = -\frac{L}{4\pi GM}\left(1 - \frac{\Omega_{\rm eq}^2}{\Omega_k^2}\right)^{-2/3}\boldsymbol{g}_{\rm eff}\ . \qquad (2)$$

- (2): $F/g_{\rm eff}$ diverges approaching the Ω-limit[1]
- (1): $F/g_{\rm eff}$ remains constant approaching the Ω-limit

This difference is of crucial importance when seeking stellar conditions leading to the $\Omega\Gamma$-limit.

4. $\Omega\Gamma$-limit

We can look for the critical angular velocity Ω_c corresponding to the $\Omega\Gamma$-limit, namely when

$$\boldsymbol{g}_{\rm tot} = \boldsymbol{g}_{\rm eff}\left[1 - \Gamma_\Omega(\theta)\right]$$

with $\Gamma_\Omega(\theta)$ the local rotation-dependent Eddington parameter

$$\Gamma_\Omega(\theta) = \frac{\kappa(\theta)L}{4\pi cGM}\frac{\tan^2(\psi(r,\theta))}{\tan^2\theta}$$

which is maximum at the equator: criticality reached at the equator first.

- Ω_c is reached if $\Gamma_\Omega(\mathrm{eq}) = 1$
- Solution $\boldsymbol{g}_{\rm eff} = \boldsymbol{0}$ ruled out by $\beta < 1/4$

Therefore

$$\Omega_c^2 = \Omega_k^2\left[1 - \Gamma(\pi/2)^{3/2}\right]$$

- When radiative acceleration is negligible: $\Omega_c \simeq \Omega_k$
- Ω_c reduced with increasing Eddington parameter

5. Local mass-flux prescription using the ω-model

We apply our results from the ω-model to estimate the local mass-flux of a rotating star. The inferred mass-flux can be seen as a 2D generalisation of the mass loss rate from [1] original theory.

We first use the CAK basic assumptions:

- No rotation
- Point source approximation
- Constant ionization thoughout the wind

Leading to the standard CAK mass loss rate

$$\dot{M} = 4\pi R^2\left(\frac{\kappa_e}{c}\right)^{\frac{1-\alpha}{\alpha}}\frac{\alpha}{c^2(1-\alpha)}\overline{Q}^{\frac{1-\alpha}{\alpha}}\left(g - \frac{\kappa_e F}{c}\right)^{\frac{\alpha-1}{\alpha}}F^{1/\alpha}$$

where κ_e is the free-electron scattering opacity, α is the CAK power index and $\overline{Q}$ is the dimensionless line-strength parameter.

- Assuming that the mass-flux by surface element from rotating stars follows the same scaling relation

$$\dot{m}(\theta) = \left(\frac{\kappa_e}{c}\right)^{\frac{1-\alpha}{\alpha}}\frac{\alpha}{c^2(1-\alpha)}\overline{Q}^{\frac{1-\alpha}{\alpha}}\left(g_{\rm eff}(\theta) - \frac{\kappa_e F(\theta)}{c}\right)^{\frac{\alpha-1}{\alpha}}F(\theta)^{1/\alpha}$$

where $F(\theta)$ can be calculated self-consistently with ESTER's full 2D models, or with the ω-model.

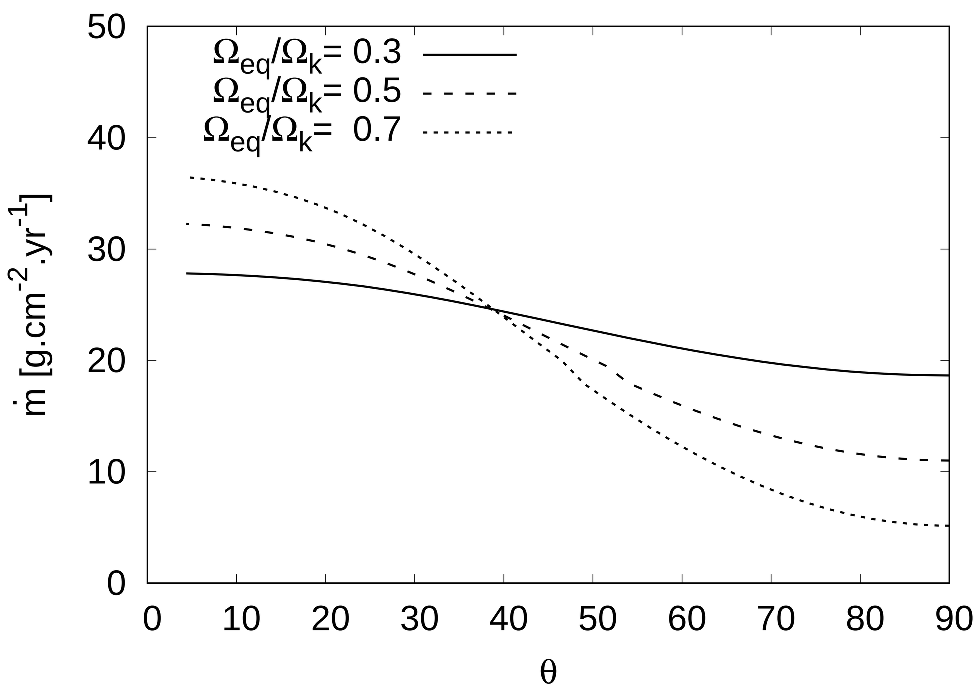

Figure 1: *Mass-flux as a function of co-latitude for a $15M_\odot$ star at ZAMS, with $Z = 0.02$ and for various rotation rates*

- The mass-flux is θ-dependent and testifies to an anisotropic stellar wind
- Polar ejection favoured: reduced loss of angular momentum
- $\overline{Q}$ and α are also θ-dependent
- Rotation-induced bi-stability jumps can dramatically increase $\dot{m}$ in the equatorial region (thus increasing the total loss of angular momentum)

6. Conclusion

- Stars more massive than $\sim 7M_\odot$ are subject to radiative winds
- Rapidly rotating stars can reach the $\Omega\Gamma$-limit associated with a critical angular velocity Ω_c
- $\Omega_c < \Omega_k$ and is reduced with increasing Γ
- The von Zeipel theorem should not be used for rapidly rotating stars
- Therefore it should not be used to derive Ω_c and $\Gamma_\Omega(\theta)$
- Rotation also induces anisotropic mass loss
- The mass-flux is favoured in polar regions if we ignore rotation-induced bi-stability jumps
- The effects of anisotropic winds are essential to appreciate the dynamical evolution of massive stars

References

[1] J. I. Castor, D. C. Abbott, and R. I. Klein. *ApJ*, 195:157–174, Jan. 1975.

[2] X. Che, J. D. Monnier, M. Zhao, E. Pedretti, N. Thureau, A. Mérand, T. ten Brummelaar, H. McAlister, S. T. Ridgway, N. Turner, J. Sturmann, and L. Sturmann. . *ApJ*, 732:68, 2011.

[3] A. Domiciano de Souza, P. Kervella, D. Moser Faes, G. Dalla Vedova, A. Mérand, J.-B. Le Bouquin, F. Espinosa Lara, M. Rieutord, P. Bendjoya, A. C. Carciofi, M. Hadjara, F. Millour, and F. Vakili. I. *A&A*, 569:A10, 2014.

[4] F. Espinosa Lara and M. Rieutord. *A&A*, 533:A43, 2011.

[5] A. Maeder. *A&A*, 347:185–193, 1999.

[6] A. Maeder and G. Meynet. *A&A*, 361:159–166, 2000.

[7] J. D. Monnier, M. Zhao, E. Pedretti, N. Thureau, M. Ireland, P. Muirhead, J.-P. Berger, R. Millan-Gabet, G. Van Belle, T. ten Brummelaar, H. McAlister, S. Ridgway, N. Turner, L. Sturmann, J. Sturmann, and D. Berger. *Science*, 317:342–, 2007.

[8] H. von Zeipel. *MNRAS*, 84:665–683, 1924.

[9] M. Zhao, J. D. Monnier, E. Pedretti, N. Thureau, A. Mérand, T. ten Brummelaar, H. McAlister, S. T. Ridgway, N. Turner, J. Sturmann, L. Sturmann, P. J. Goldfinger, and C. Farrington. *ApJ*, 701:209–224, 2009.

[1] Ω-limit: $\Omega_{\rm eq} = \Omega_k$