Application of the methods of quantum chaos to the oscillations of rapidly rotating stars

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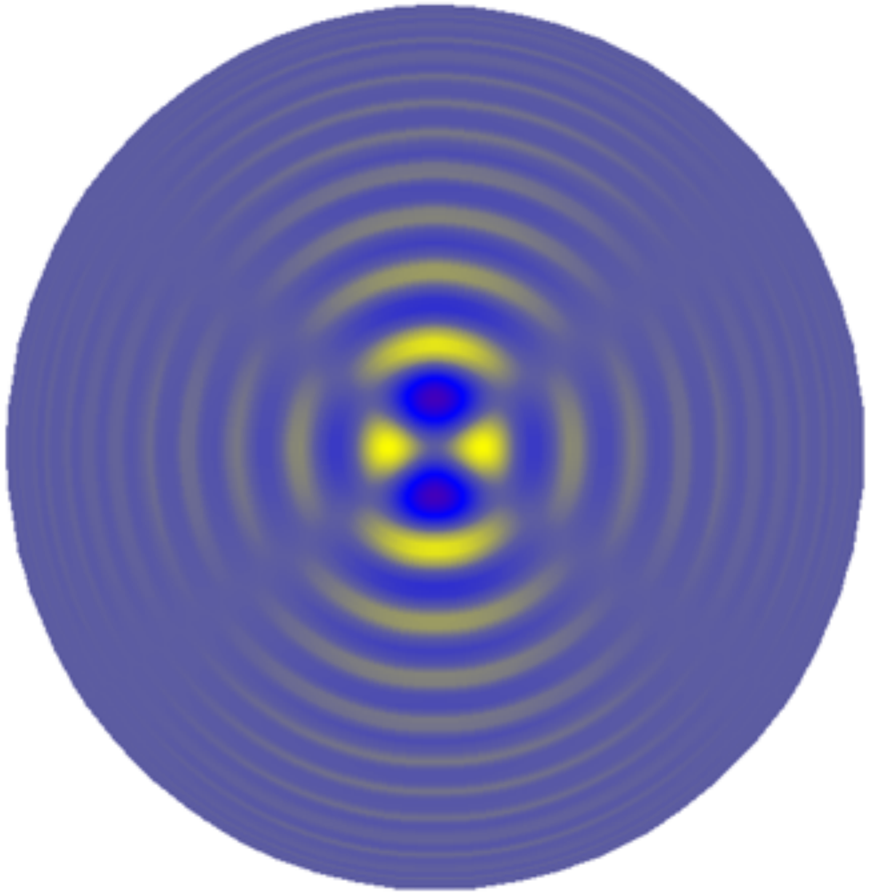
IRAP - LPT

with François Lignières et Bertrand Georgeot

Asteroseismology

- Identify oscillation modes
- Find the frequency spectrum
- Constrain the stellar interior

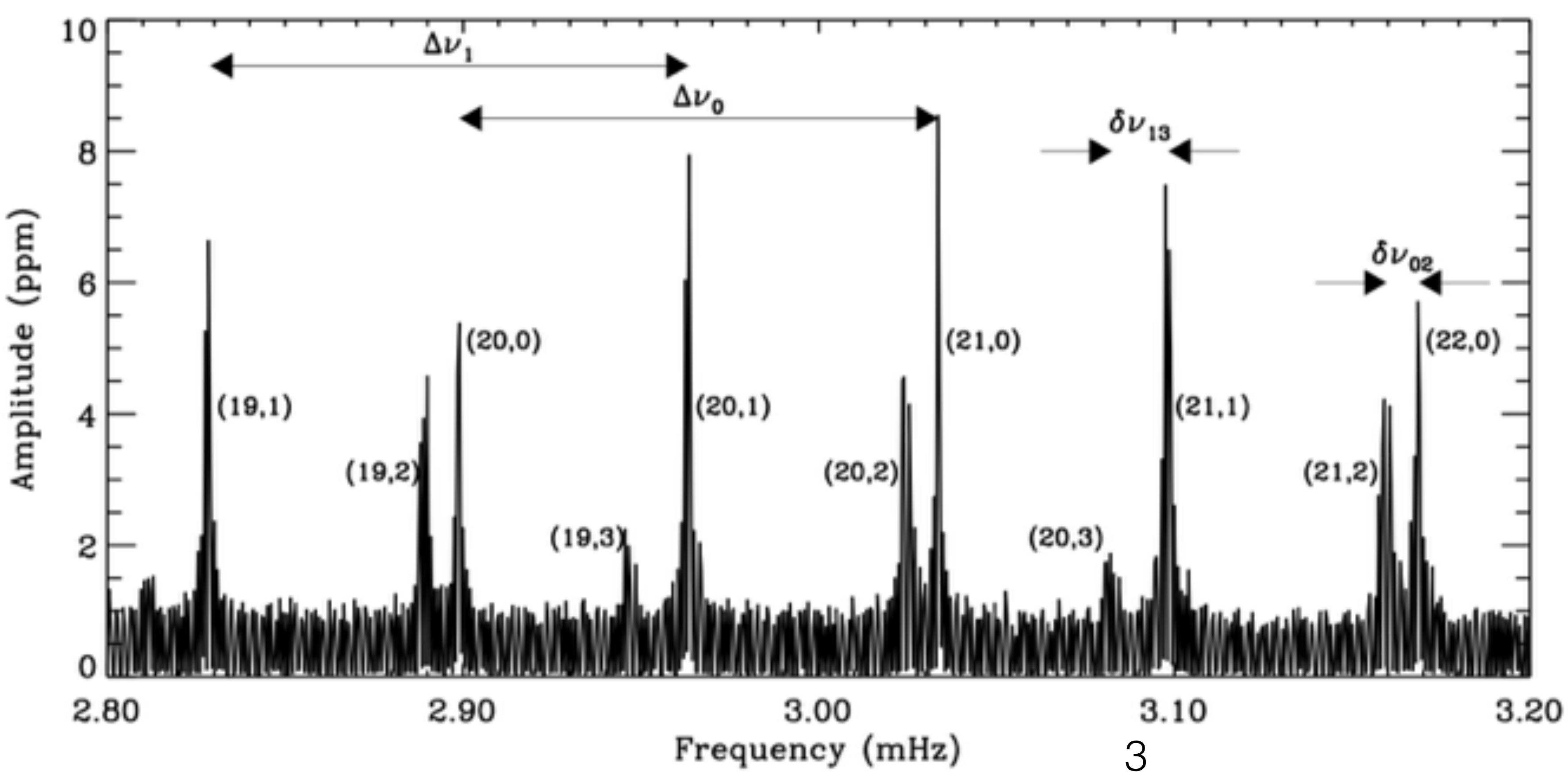


n, ℓ

Slowly rotating stars

Tassoul's asymptotic formula : $\omega_{n,l} \simeq \Delta\omega \left(n + \frac{\ell}{2} + \frac{1}{4} + \alpha \right)$

$$\Delta\omega = \pi \left(\int_0^{R_e} \frac{\mathrm{d}r}{c_s} \right)^{-1}$$

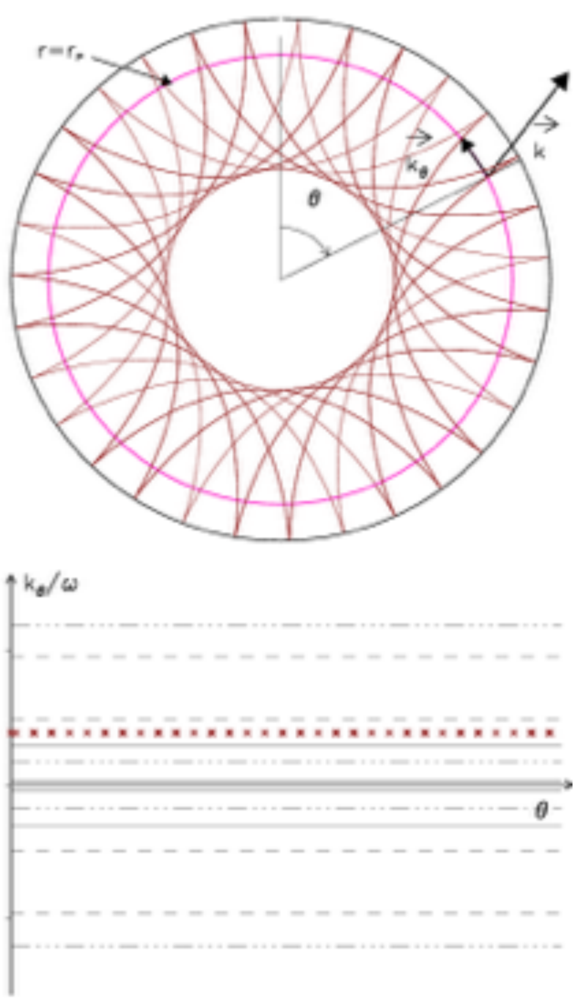


Effect of rotation

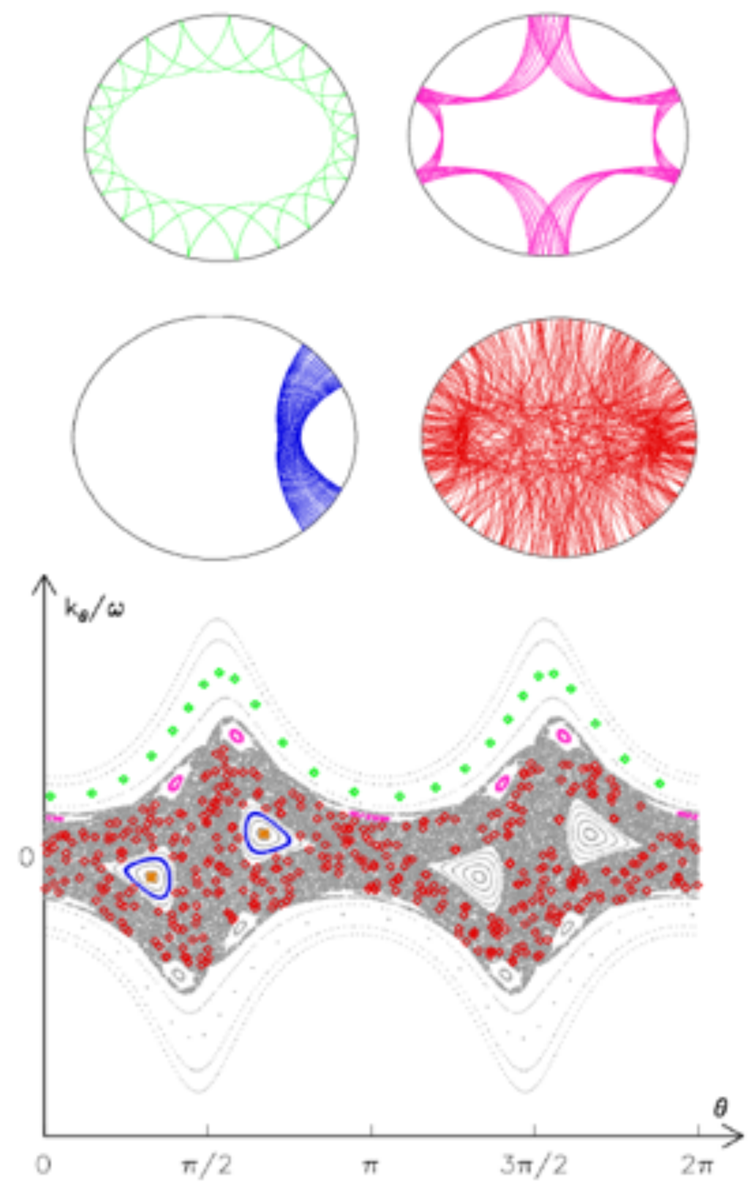
- Tassoul's formula is no longer valid
- New types oscillation modes appear

Acoustic rays

Slow rotation

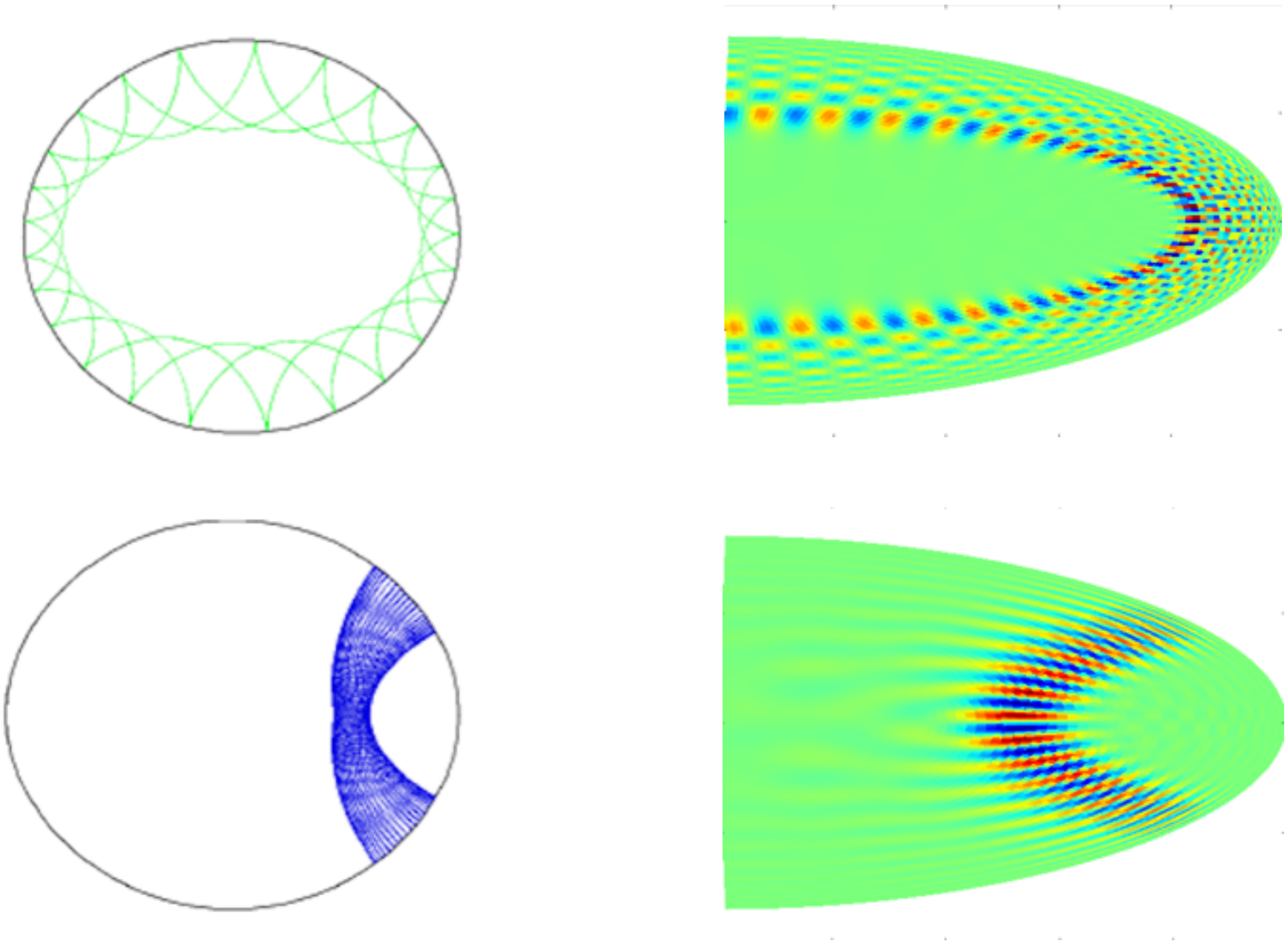


Fast rotation

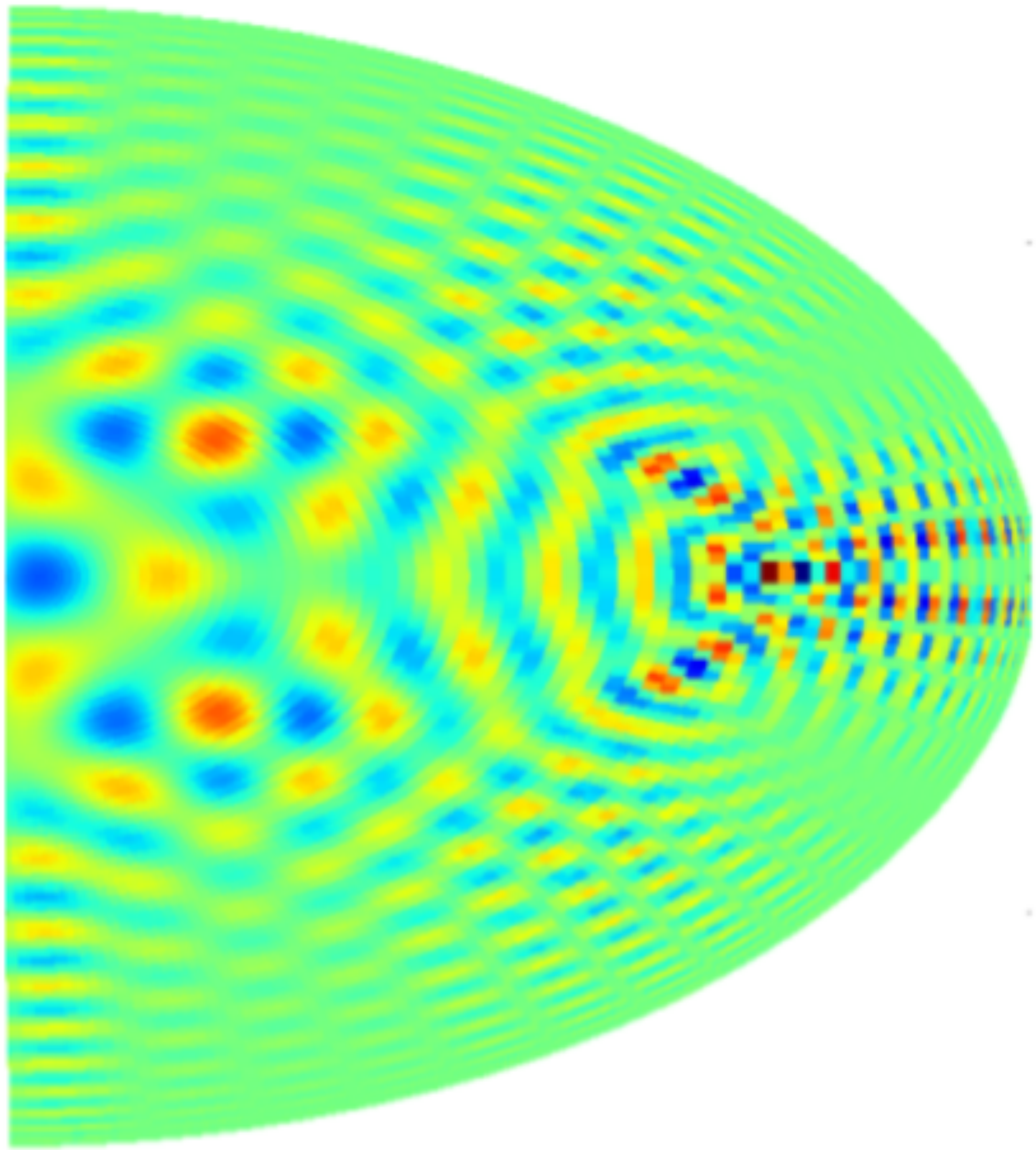


Regular modes

Oscillation modes are built on ray paths



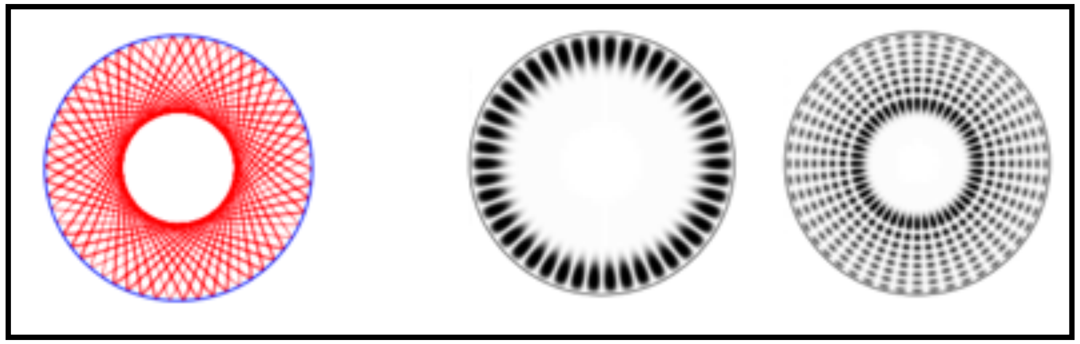
Chaotic modes



Structure of the frequency spectrum ?

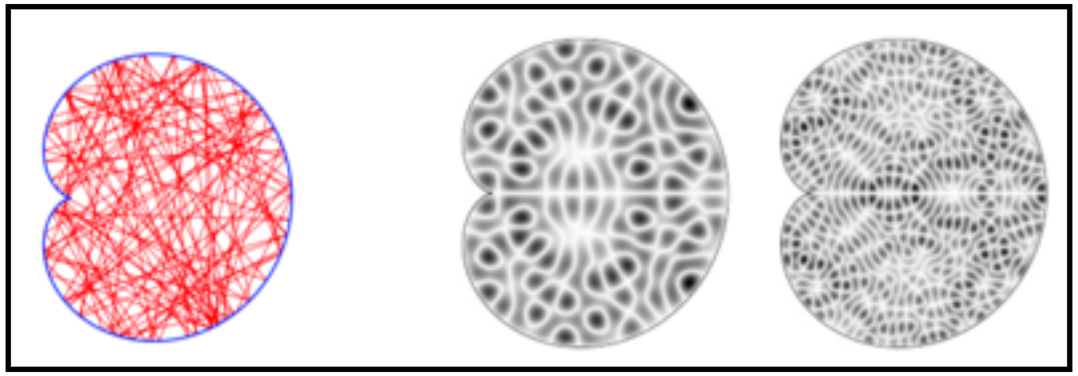
Asymptotic formula for chaotic modes ?

Integrable dynamics



Quantization

Chaotic dynamics



Energy levels statistics

Gutzwiller formula

Gutzwiller formula :

$$\mathrm{d}(E) - \overline{\mathrm{d}}(E) = \frac{1}{h} \sum_j \frac{T_j}{k_j \sqrt{\det(\boldsymbol{M}_j - \boldsymbol{I})}} \, e^{i\left(\frac{S_j(E)}{h} + \nu_j\right)}$$

Quantum

> frequency spectrum

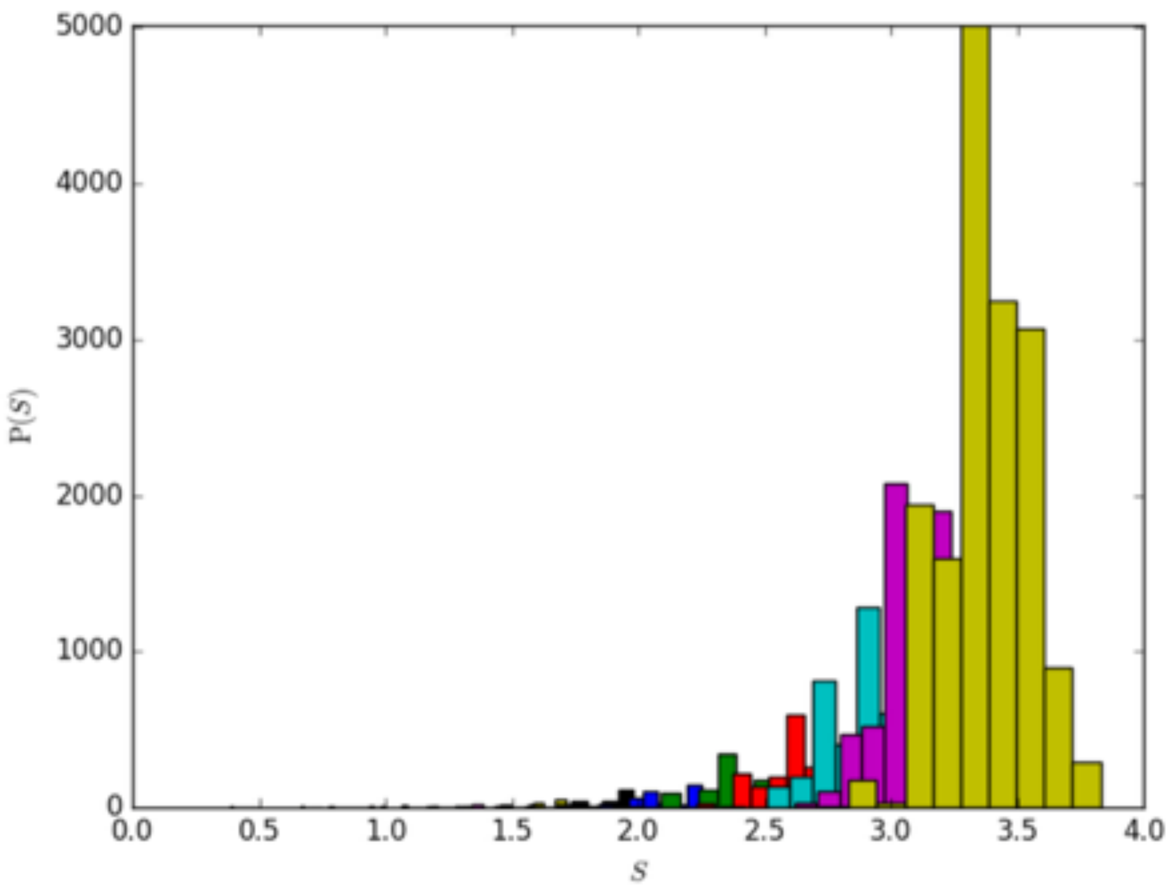
Classical

> dynamics of acoustic rays

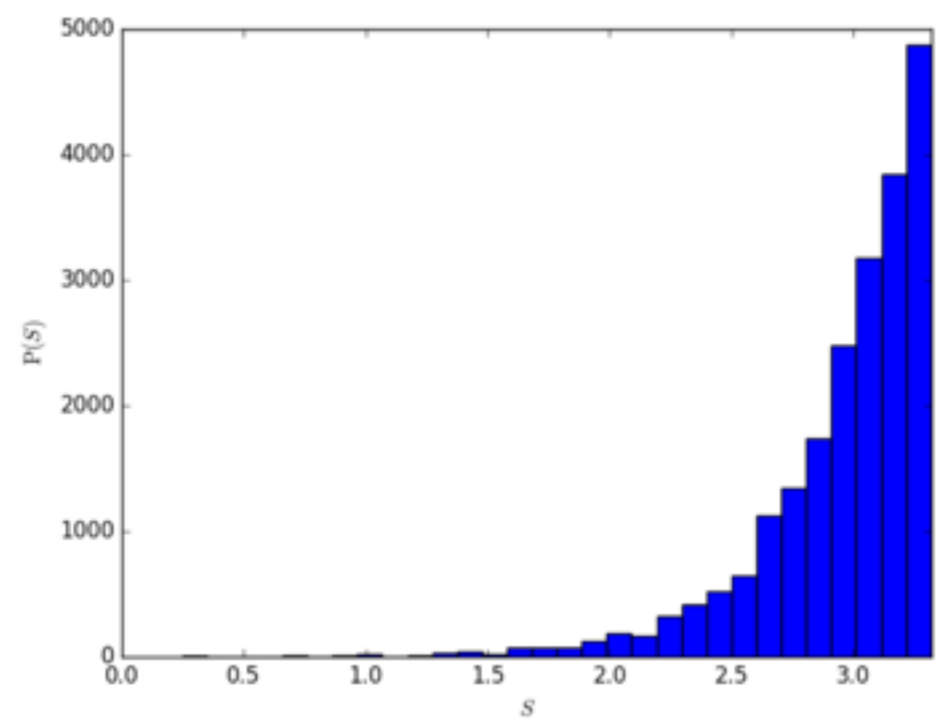
Chaotic map : actions of periodic orbits

$$p_{n+1} = p_n + \mathrm{V}_0\big(2q_n - \mathrm{sign}(q_n)\big)$$

$$q_{n+1} = p_{n+1} + q_n$$

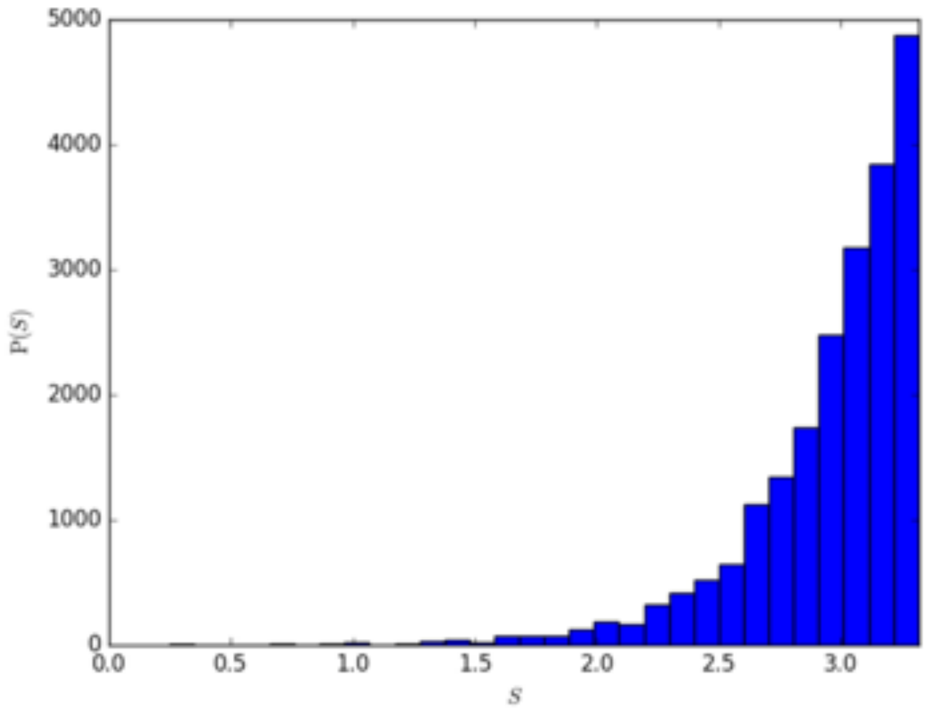


Individual distributions

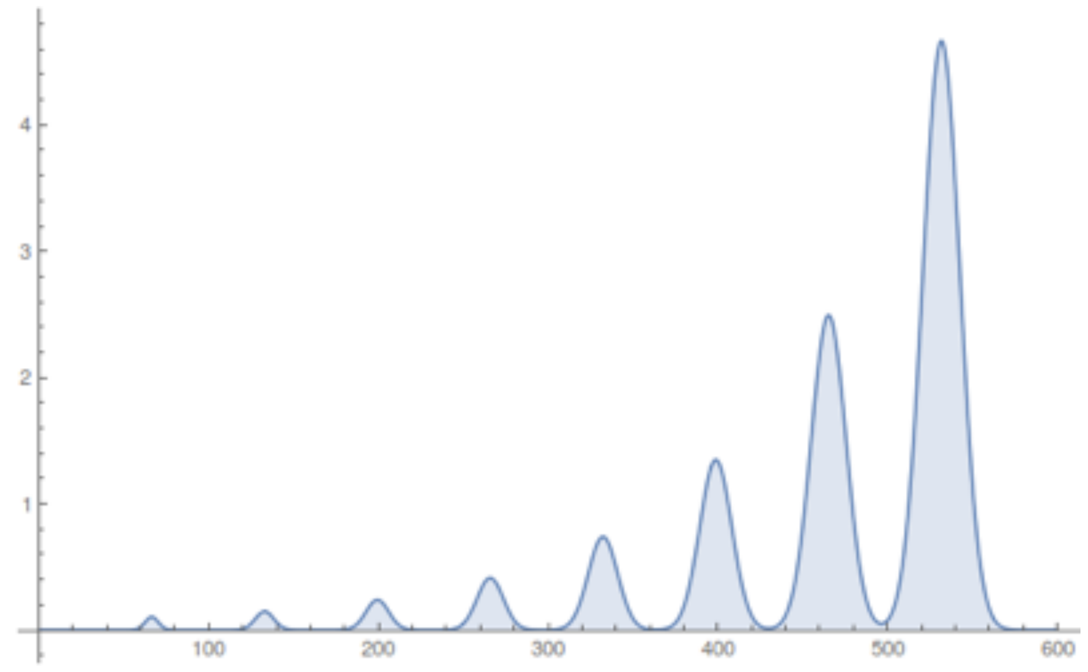


Total distribution

Oscillation in the distribution of the actions



Chaotic map



Chaotic trajectories inside a rotating star

Conclusion

- We need the tools of quantum chaos to reach a theoretical understanding of the chaotic spectrum in rapidly rotating stars
- The statistical properties of chaotic spectra are said to be universal, but we have found a system that will not follow this universality.