$\Omega\Gamma$-limit and anisotropic mass loss from rapidly rotating early-type stars

Damien Gagnier

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PHD day

Introduction

Introduction

- Rotation breaks the 1D symmetry
- Rotation induces large scale flows: transport of chemicals and angular momentum
- Rotation induces mass-loss and angular momentum loss anisotropy

- What is the break-up velocity when both rotation and radiation have significant effects on the total gravity? ($\Omega\Gamma$-limit)
- How does rotation affect mass loss from radiation-driven winds?

Critical angular velocity at the $\Omega\Gamma$-limit

Critical angular velocity at the $\Omega\Gamma$-limit

Debate on the subject:

- Langer (1997, 1998): Stars close to Γ-limit have a lower critical angular velocity
- Glatzel (1998): Eddington parameter Γ has no effect on the critical rotation
- Maeder (1999), Maeder & Meynet (2000) : 2 different critical ang. velocities :
  - $\Omega_c = \Omega_k = \sqrt{GM/R_{\mathrm{eq}}^3}$ if $\Gamma < 0.639$
  - $\Omega_c \to 0$ when $\Gamma \to 1$ and $\Gamma > 0.639$

$$\Gamma = \frac{\kappa L}{4\pi c G M}$$

The radiative flux: von Zeipel theorem revisited

- Maeder (1999), Maeder & Meynet (2000) : Use model of von Zeipel (1924)

$$\boldsymbol{F} \propto -\boldsymbol{g}_{\mathrm{eff}}^{4\beta}$$

with $\beta = 1/4$
Assuming solid body rotation

$$\boldsymbol{F} = -\frac{L(P)}{4\pi G M_*}\boldsymbol{g}_{\mathrm{eff}}$$

with $M_* = M(1 - \Omega^2/2\pi G \rho_m)$

The radiative flux: von Zeipel theorem revisited

Developing all quantities around their average on an isobar : radiative flux for shellular rotation $\Omega = \Omega(r)$

$$\boldsymbol{F} = -\frac{L(P)}{4\pi G M_*}[1 - \zeta(\theta)]\boldsymbol{g}_{\mathrm{eff}}$$

where $\zeta(\theta) \ll 1$: mild dependence with colatitude

- Only valid for slow rotators
- In contradiction with both observations and 2D-ESTER models

Interferometric observations of several rapidly rotating stars + 2D-ESTER models : If

$$\boldsymbol{F} \propto -\boldsymbol{g}_{\mathrm{eff}}^{4\beta}$$

then $\beta < 1/4$

The radiative flux: ω-model (Espinosa Lara & Rieutord (2011)

- This model assumes $\boldsymbol{F} = -f(r,\theta)\boldsymbol{g}_{\rm eff}$: $\boldsymbol{F}$ and $\boldsymbol{g}_{\rm eff}$ antiparallel
- In the envelope of a star: flux conservation $\boldsymbol{\nabla}\cdot\boldsymbol{F} = 0$
- Equation for $f(r,\theta)$ can be solved analytically

$$f(r,\theta) = \frac{L}{4\pi GM}\frac{\tan^2\psi(r,\theta)}{\tan^2\theta}$$

where $\psi(r,\theta)$ is obtained solving

$$\cos\psi + \ln\tan(\psi/2) = \frac{1}{3}\omega^2 r^3\cos^3\theta + \cos\theta + \ln\tan(\theta/2)$$

The radiative flux: ω-model (Espinosa Lara & Rieutord (2011)

At the equator this can be solved analytically and yields

$$\boldsymbol{F}(R_{\rm eq}, \pi/2) = -\frac{L}{4\pi GM}\left(1 - \frac{\Omega_{\rm eq}^2}{\Omega_k^2}\right)^{-2/3} \boldsymbol{g}_{\rm eff}$$

- $F/g_{\rm eff}$ diverges at Ω-limit while it does not with vZ theorem
- Difference crucial when seeking conditions leading to ΩΓ-limit

Critical angular velocity at the ΩΓ-limit

ΩΓ-limit : somewhere at the stellar surface

$$\boldsymbol{g}_{\rm tot} = \boldsymbol{g}_{\rm eff} + \boldsymbol{g}_{\rm rad} = \boldsymbol{0} \tag{1}$$

We introduce the limiting flux

$$\boldsymbol{F}_{\rm lim}(\theta) = \frac{-c}{\kappa}\boldsymbol{g}_{\rm eff}(\theta)$$

($\boldsymbol{g}_{\rm eff} = -\boldsymbol{g}_{\rm rad} = -\frac{\kappa \boldsymbol{F}_{\rm lim}}{c}$) We introduce the rotation-dependent Eddington parameter $\Gamma_\Omega(\theta)$

$$\Gamma_\Omega(\theta) = \frac{F(\theta)}{F_{\rm lim}(\theta)} = \frac{\kappa(\theta)L}{4\pi cGM}\frac{\tan^2(\psi(r,\theta))}{\tan^2\theta}$$

(1) can thus be written

$$\boldsymbol{g}_{\rm tot} = \boldsymbol{g}_{\rm eff}[1 - \Gamma_\Omega(\theta)] = \boldsymbol{0} \tag{2}$$

Critical angular velocity at the ΩΓ-limit

$$\boldsymbol{g}_{\mathrm{tot}} = \boldsymbol{g}_{\mathrm{eff}}[1 - \Gamma_{\Omega}(\theta)] = \mathbf{0}$$

- $\boldsymbol{g}_{\mathrm{tot}} = \mathbf{0}$ if there is a colatitude where $\Gamma_{\Omega}(\theta) = 1$
- Maeder & Meynet (2000) also considered $\boldsymbol{g}_{\mathrm{eff}} = \mathbf{0}$ as a solution
- But $\beta < 1/4$ rules out this solution
- (Can only be reached going through a non-realistic gravitationally unbound stage of supercritical angular velocity)

$\Gamma_{\Omega}(\theta)$ is an increasing function of θ :

- $f(r, \theta)$ always increases with increasing θ
- $\kappa(\theta)$ increases as temperature decreases

$\longrightarrow$ **Criticality always reached at equator first**

Critical angular velocity at the $\Omega\Gamma$-limit

The condition giving the critical angular velocity Ω_c is :

$$\Gamma_\Omega(\pi/2) = \frac{\kappa(\pi/2)L}{4\pi cGM}\left(1 - \frac{\Omega_c^2 R_{\rm eq}^3}{GM}\right)^{-2/3} = 1$$

which gives

$$\Omega_c^2 = \Omega_k^2\left[1 - \Gamma_{\rm max}^{3/2}\right] ,$$

where

$$\Gamma_{\rm max} = \Gamma(\pi/2) = \frac{\kappa(\pi/2)L}{4\pi cGM}$$

is the standard Eddington parameter.

$\longrightarrow$ Ω_c **is reduced with increasing** Γ

The effects of rotation on stellar winds

The effects of rotation on stellar winds

The interaction of mass loss and rotation is manifold :

- Latitudinal dependence of mass loss for a given rotation
- Change of the global mass loss with rotation
- Anisotropic mass loss induces anisotropic loss of angular momentum
- Anisotropic mass loss may contribute to the driving or damping of meridional circulation

Local mass and angular momentum fluxes

Local mass-flux :

$$\dot{m}(\theta) = \frac{\alpha}{v_{\rm th}(\theta)c}\left(\frac{k}{1+\alpha}\right)^{1/\alpha'}\left[\frac{c}{\kappa_e(1-\alpha)}\left(g_{\rm eff}(\theta) - \frac{\kappa_e F(\theta)}{c}\right)\right]^{\frac{\alpha'-1}{\alpha'}} F^{1/\alpha'}(\theta)$$

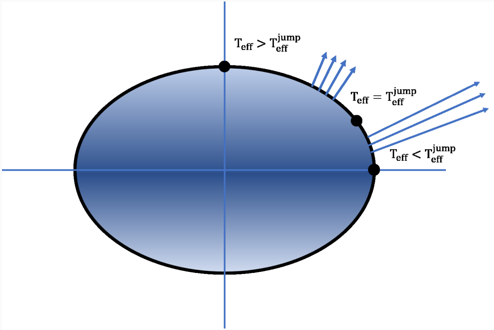
- $\alpha' = \alpha - \delta$
- α, k, δ : force multiplier parameters, depend on $T_{\rm eff}$
- $v_{\rm th}$ thermal velocity (usually $v_{\rm th} = v_{\rm th}(H)$)
- κ_e : electron scattering opacity

$\longrightarrow$ **Anisotropic mass loss, favoured polar ejection at first glance**

Local loss of angular momentum :

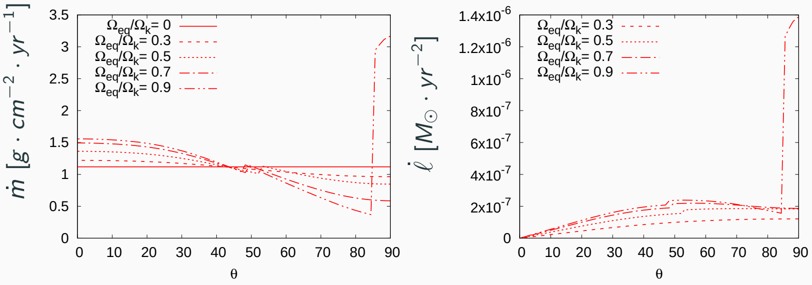
$$\dot{\ell}(\theta) = \dot{m}(\theta)\Omega(\theta)R^2(\theta)\sin^2(\theta)$$

Bi-stability jumps



- $T_{\rm eff}^{\rm jump}$: effective temperature at which Fe IV $\rightarrow$ Fe III which has a stronger line acceleration in the lower wind
- When $\exists T_{\rm eff}(\rm eq) < T_{\rm eff}^{\rm jump} < T_{\rm eff}(\rm pole)$, $\dot{M}$ increases in the equatorial region

The effects of rotation on stellar winds



- SWR : mass loss favoured in polar regions → **low AM loss**
- TWR : bi-stability limit is crossed, mass loss favoured between $\theta_{\rm jump}$ and the equator → **significant loss of AM**

Conclusion

The effects of rotation on stellar winds

- von Zeipel theorem should not be used to seek conditions for critical rotation
- ω-model in agreement with 2D ESTER models & observations
- **Unique critical angular velocity** at the $\Omega\Gamma$-limit
- Ω_c **decreases with increasing** Γ

- Rotation induces **anisotropic mass loss and angular momentum loss**
- **SWR** : mass-flux favoured in polar regions & low angular momentum loss
- **TWR** : mass-flux favoured in equatorial region & high angular momentum loss
- **Anisotropy & rotation-induced bi-stability jump** effects on the wind are crucial to appreciate dynamical evolution of massive stars

Questions?